

## A FELICITOUS FRAGMENT OF THE PREDICATE CALCULUS

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A notation without variables of quantification is provided for a fragment of the lower predicate calculus with one- and two-place predicates. It can be translated nearly symbol-by-symbol into reasonable English. The fragment is decidable. An apparently minor extension giving flexibility of quantifier scope yields a fragment that is undecidable.

Horses are animals, but not conversely. Therefore, heads of horses are heads of animals, though not conversely. But lovers of horses, if by this we mean lovers of all horses, are not necessarily lovers of animals—that is, of all animals—since they may fail to love crocodiles: in this case, it is the converse that is true, namely, that lovers of all animals are lovers of all horses. Likewise, lovers of (all) heads of animals are lovers of (all) heads of horses, and heads of lovers of (all) animals are heads of lovers of (all) horses, but in neither case conversely; and admirers of (all) lovers of (all) horses are admirers of (all) lovers of (all) animals, but not conversely. The example, in its earlier stages, is from De Morgan ([1], p. 131) and examples like it are to be found earlier in Junge [4] in discussion of the topics of genus and species. The general rule of validity for our extension of it is that the complex term containing “horses” must be in subject-position if the number of occurrences of “all” in it (and in its counterpart containing “animals”) is even, and in predicate-position if this number is odd.

The English expression of these consequences is more compact and perspicuous, provided some care is taken with quantifiers, than their predicate calculus equivalents. For example, the statement that admirers of all lovers of all horses are admirers of all lovers of all animals has the form

$$\forall x(\forall y(\forall z(az \supset gyz) \supset fxy) \supset \forall y(\forall z(bz \supset gyz) \supset fxy)) \quad (1)$$

The English expression contains no counterpart of variables of quantification. The expedient is suggested of introducing quantifiers with the grammatical function of what are called in older grammar-books *pronoun-relatives* (not to be confused with relative pronouns) like Latin *quicumque*.

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