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A STRONGER DEFINITION OF A RECURSIVELY INFINITE SET

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1 Introduction. The purpose of this announcement is to strengthen the definition of a recursively infinite set as defined by Dekker and Myhill in [2]. This can be done after we have proved that any function that maps an immune set, α, one-to-one into itself and has a partial recursive extension must be an ω-permutation of α.

2 Preliminaries. Let ε stand for the set of nonnegative integers (*numbers*), V for the class of all subcollections of ε (*sets*), and $\mathcal{F}$ for the set of all mappings from a subset of ε into ε (*functions*). If f is a function, we write δf and ρf for its domain and range respectively. The relation of inclusion is denoted by $\subset$ and that of proper inclusion by $\subsetneq$. Certain families of functions are denoted by special symbols.

$$\mathcal{F}_{1-1} = \{f \in \mathcal{F} \mid f \text{ is one-to-one}\},$$
$$\mathcal{A} = \{f \in \mathcal{F} \mid f \text{ has a partial recursive extension}\},$$
$$\mathcal{A}_{1-1} = \{f \in \mathcal{A} \mid f \text{ has a one-to-one partial recursive extension}\}.$$

The sets α and β are *recursively equivalent* [written: $\alpha \simeq \beta$], if $\delta f = \alpha$ and $\rho f = \beta$, for some $f \in \mathcal{A}_{1-1}$.

We recall from [1], Proposition 1 that

$$(*) \qquad f \in \mathcal{A}_{1-1} \Leftrightarrow f, f^{-1} \in \mathcal{A}, \text{ for } f \in \mathcal{F}_{1-1}.$$

A permutation of a set α is an ω-permutation, if $f \in \mathcal{A}_{1-1}$. The reader is assumed to be familiar with the contents of [2].

3 Main Results.

Notation. For $f \in \mathcal{F}$, f^n is defined for $n \in \varepsilon$, as follows: $f^0 = i$, where i is the identity function, and $f^{n+1} = f \circ f^n$, where $\circ$ is function composition, and f^{n+1} has the appropriate domain.

Theorem 1. *Let α be an immune set and $f \in \mathcal{F}_{1-1} \cap \mathcal{A}$ such that $\delta f = \alpha$ and $\rho f \subset \alpha$, then f is an ω-permutation of α.*

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