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ON GENERATING THE FINITELY SATISFIABLE FORMULAS

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Results by B. Trahtenbrot [4] and Th. Hailperin [3] show that the class of finitely valid formulas of first-order predicate logic is not recursively enumerable. In this paper we shall present a system[1] which generates all formulas of first-order predicate logic which are satisfiable in some non-empty finite individual domain. The system forms a kind of calculus for finite satisfiability in the sense that the finitely satisfiable formulas are all obtainable from certain "axioms" by means of several "rules of inference." As in [3] and [4], we shall consider only formulas of the first-order predicate logic without identity. A forthcoming paper will deal with formulas of first-order predicate logic with identity.

The primitive symbols of our first-order predicate logic are the (individual) variables $v_1, v_2, \ldots$, the (individual) constants $c_1, c_2, \ldots$, for each $n \geq 0$ the n-ary predicates $p_1^n, p_2^n, \ldots$, the logical symbols $\neg$, $\wedge$ and $\forall$, and the parentheses (,). Atomic formulas and formulas as well as the concepts of free and bound occurrences of variables in formulas are defined in the customary way, and all occurrences of constants in formulas will be considered free occurrences. If each of $t_1, \ldots, t_n$ is a term (that is, a variable or constant) and each of $t_1', \ldots, t_n'$ is a variable not occurring in the formula φ or is a constant, we denote by $\varphi[t_1/t_1', \ldots, t_n/t_n']$ the formula obtained by replacing each free occurrence of t_i in φ by t_i'.

The model-theoretic notions used here are as in J. L. Bell and A. B. Slomson ([1] pp. 55-56). In particular, if $\mathfrak{A}$ is a model structure, we denote the set on which $\mathfrak{A}$ is defined by $|\mathfrak{A}|$, and the element of $|\mathfrak{A}|$ to which a constant d is assigned by $\mathfrak{A}$ we denote by $d^{\mathfrak{A}}$. We write $\mathfrak{A} \models \varphi[a_1, \ldots, a_m]$ if the assignment of $a_i \in |\mathfrak{A}|$ to v_i satisfies φ, where m is the greatest integer such that v_m occurs free in φ. If there is a model structure $\mathfrak{A}$ and $a_1, \ldots, a_m \in |\mathfrak{A}|$ such that $\mathfrak{A} \models \varphi[a_1, \ldots, a_m]$, then φ is *satisfiable,* and if $|\mathfrak{A}|$ has exactly k elements, φ is *k-satisfiable.* A formula is *finitely satisfiable* if it is k-satisfiable for some positive integer k.

1. A description of this system was first presented at the IV'th International Congress for Logic, Methodology and Philosophy of Science; see [2].

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