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A SIMPLE ALGEBRA OF FIRST ORDER LOGIC

CHARLES C. PINTER

1. *Introduction*[1] The idea of making algebra out of logic is not a new one. In the middle of the last century George Boole investigated a class of algebras, subsequently named Boolean algebras, which arose naturally as a way of algebraizing the propositional calculus. More recently there have appeared several algebraizations of the first-order predicate calculus, of which the most important are the polyadic algebras of Halmos [3], and the cylindric algebras of Tarski [5]. Each of these two approaches to algebraic logic has its relative merits, and presents conceptual difficulties which have proved to be a stumbling block for many an interested reader.

The purpose of this paper is to present a formulation of algebraic logic which is closely related to both polyadic and cylindric algebras and is, in a sense, intermediate between the two. The advantage of the system we are about to present is that it is based upon a small number of axioms which are extremely simple and well motivated. From a didactic point of view, this may be the most satisfactory way of introducing the student and non-specialist to the ideas and methods of algebraic logic. We will show precisely how our algebra is related to cylindric and polyadic algebras.

2. *Quantifier algebras* In this section we introduce a class of algebras to be called *quantifier algebras*,[2] which may be viewed as an algebraization of the first-order predicate calculus without equality. We begin by examining a special class of quantifier algebras, called quantifier algebras of formulas. The construction of these algebras has a metalogical character and extends the well-known method for constructing Boolean algebras from the propositional calculus.

Let Λ be a first-order language with a sequence $\langle v_\kappa \rangle_{\kappa<\alpha}$ of variables, and let θ be a theory of Λ. We let $\mathsf{Fm}^{(\Lambda)}$ designate the set of all the formulas of Λ, and $\mathsf{Fm}^{(\Lambda)}/{\equiv_\theta}$, the preceding set modulo the relation $F \equiv_\theta G$

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2. The term *quantifier algebra* has been used by several authors in different senses, all differing from the present one.

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