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SENTENTIAL CALCULUS FOR LOGICAL FALSEHOODS

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Several axiomatic systems for sentential calculus have been developed. Such systems are generally motivated by a consideration of logically true sentences of the formal language. In this paper I present a finitely axiomatized system of sentential calculus for logically false sentences.

1. *Introduction.* Consider a formal language L with the following symbols:

Sentential variables: $P_1, P_2, \ldots$
Sentential connectives: &—"and," $\vee$—"or," $\neg$—"not"
Punctuation:), and (

I will assume the standard definition for "sentence of L." The meta-symbols $R, R_1, R_2, \ldots$ will be used to refer to sentences of L. In addition, I will presuppose the standard theory of two-valued truth tables. I will say that a sentence of L is logically true (**LT**) if and only if the final column of its truth table has only **T**'s. I will say that a sentence of L is logically false (**LF**) if and only if the final column of its truth table has only **F**'s. I will say that two sentences R_1 and R_2 of L are logically equivalent (R_1 **LE** R_2) if and only if the sentence $(R_1 \& R_2) \vee (\neg R_1 \& \neg R_2)$ is **LT**.

2. *The System* **SCT**. In [1], Hilbert and Ackermann present an axiomatic system of sentential calculus for logical truths. With some small notational differences, their system uses the symbols mentioned above and in addition the symbol "$\rightarrow$". As they note, however, this symbol is to be considered an abbreviation; if R_1 and R_2 are any two sentences of the language, then $R_1 \rightarrow R_2$ is to be considered an abbreviation for the sentence $\neg R_1 \vee R_2$ ([1], pp. 27-28). In discussing their system, I will eliminate this abbreviation. Since their system is primarily concerned with **LT** sentences, I will refer to their system as **SCT** (sentential calculus for truths). With slight notational differences and the removal of the symbol "$\rightarrow$", the Hilbert and Ackermann system may be presented as follows:

Axioms:

(ta) $\neg(P_1 \vee P_1) \vee P_1$

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