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A NOTE ON TRANSITIVITY

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Prefixing *CCpqCCrpCrq*[1] (**T pre**) and suffixing *CCpqCCqrCpr* (**T suf**) are usually taken to be the two theorem forms of transitivity, chiefly because in the presence of the rules of substitution and detachment, they both yield the derived rule of transitivity "*From Cpq and Cqr infer Cpr.*" Because of this, even though Sobociński [4] reports a result attributed to Łukasiewicz that **T pre** and **T suf** are mutually independent, one might be tempted to suppose that in any reasonable context they are equivalent (interreplaceable) forms of the same basic idea-transitivity. One might suppose this even given Sobociński's proof[2] for matrix $\mathfrak{A}$, which he uses to show the independence of **T pre** from **T suf**, does not even satisfy identity *Cpp* (**T** identity); but such is *not* the case.

To be sure, there are contexts in which they are interreplaceable, e.g., in the presence of permutation "*From CpCqr infer CqCpr*" (**DR perm**), or restricted permutation "*From CpCCqrs infer CCqrCps*" (**DR rest perm**), or as Sobociński shows in the presence of unrestricted assertion *CpCCpqq* (**T** assertion).[3] A closer inspection of **T pre** and **T suf** reveals that there is indeed some permutation already present in **T suf**, i.e., in the consequent *q* precedes *p*, which is not the case in **T pre**. This suggests **T suf** is a more powerful (useful) form of transitivity. An example of this may be taken from Anderson's pure calculus of entailment E_I [1] where one formulation (here denoted by E_I1) has the following axioms together with the rules of substitution and detachment:

E_I1 Ax1. *CCCppqq*
E_I1 Ax2. *CCpqCCqrCpr*
E_I1 Ax3. *CCpCpqCpq*

This formulation is essentially the formulation I_2 of Anderson, Belnap and

1. The symbolism of J. Łukasiewicz is used throughout, *cf.* [3], pp. 77-83. The names of theorems and rules follow Anderson and Belnap, *cf.* [1], p. 42.

2. *Op. cit.*, p. 49.

3. *Ibid.*

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