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LOCAL RECURSIVE THEORY

VLADETA VUČKOVIĆ

1 *Introduction.* The purpose of this paper is to outline a generalization of the recursive theory, which is quite different from existing generalizations. Instead of an axiomatic treatment as an ultimate goal, we proceed in the opposite direction, considering sets which are, as far as recursive notions are in question, well behaved only locally. Our hope is that such a point of view will not end in an imitation of the recursive theory and that it will produce new, interesting and non-trivial problems and results.

In order to give a definite picture of such a *Local Recursive Theory* we do not present the most general case possible. Already in this case the number of problems which arise is overwhelming.

Methodically, local recursive theory is a development of Malcev's general theory of enumerations. However, in this paper we use only the simplest enumerations, *indexings*, i.e., bijective maps $\alpha : N \rightarrow U_\alpha$, where N is the set of non-negative integers and U_α a denumerable set.

2 *Recursive Manifolds.* If $\alpha : N \rightarrow U_\alpha$ is an indexing, we can identify U_α with N and pursue the recursive theory on U_α in a trivial way. However, if $\mathfrak{A}$ is a family of indexings $\alpha : N \rightarrow U_\alpha$ and $M = \bigcup_{\alpha \epsilon \mathfrak{A}} U_\alpha$, the introduction of recursive notions on M, by use of sets U_α, becomes a problem whose outcome is not obvious.

Definition 2.1. A set M is called a *recursive manifold* (an RM) iff:

(i) There is a family $\mathfrak{A}$ (an *atlas* on M) of indexings $\alpha : N \rightarrow U_\alpha$, where each U_α is a subset of M (a *local neighborhood*), such that $M = \bigcup_{\alpha \epsilon \mathfrak{A}} U_\alpha$;

(ii) For all pairs $\langle \alpha, \beta \rangle \epsilon \mathfrak{A}^2$, the numerical map $\alpha^{-1} \circ \beta$ is a partial recursive function with recursive domain (inclusive $\emptyset$, the empty set, as a possible domain).

Example 2.1. Let M be an infinite set and $\alpha : N \rightarrow U$ an indexing of a subset U of M. If $M = U$, M is an RM with atlas $\{\alpha\}$. If $M \neq U$, to every $x \epsilon M - U$ correspond the local neighborhood $U_x = U \cup \{x\}$ and the indexing $\alpha_x : N \rightarrow U_x$, defined by

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