

A SIMPLIFIED SEMANTICS FOR MODAL LOGIC

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1. Until recently, philosophers could object to modal logic on the grounds that there were no known semantics for the many modal calculuses. To be sure, there were some isolated interpretations for individual systems, but there was no general theory which would apply to the many different systems, and which would give an account, from a semantical point of view, of the relations between them. With the work of Kripke, Hintikka, Kanger, Lemmon, and others, that has, of course, changed.

Kripke's semantics for modal logic begins with the idea of a *model structure*, $\langle G, K, R \rangle$, where K is a nonempty set (intuitively, of possible worlds), R is a relation on K (a relation of relative possibility or alternativeness), and $G \in K$. Truth-functional propositions are evaluated with respect to possible worlds (members of K) in the usual ways, and propositions of the sort, *necessarily B*, are said to be true in a world H in K if and only if B is true in every world, H' , in K such that HRH' . A formula is valid if it is true in G for every appropriate model structure, $\langle G, K, R \rangle$. By stipulating different properties for R (e.g. reflexivity, reflexivity and transitivity, etc.) different model structures are defined which validate different classes of formulas corresponding to the different modal calculuses. Thus, e.g., all and only formulas provable in $S4$ are valid in all model structures in which R is both reflexive and transitive. With some minor modifications this account can be generalized to provide semantics for most all the standard systems of modal logic. (Cf. [11] and [12].) Similar moves are made by Hintikka in [8] and Kanger [9]. Lemmon develops analogous devices in his algebraic semantics for the systems in [13] and [14].

Nevertheless, while the mathematical problem of developing an adequate semantical theory of the modalities may have been solved in this way, philosophers critical of modal logic might still object that since these accounts depend on a notion of a possible world, which is as obscure as the original concepts of possibility and necessity, no real clarification of these modalities has been achieved by these interpretations. Moreover, these critics might also object to the introduction of the relations, R , between

Received March 1, 1970