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RADO'S THEOREM AND SOLVABILITY OF SYSTEMS OF EQUATIONS

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In this paper we consider finite or infinite systems of equations each in finitely many unknowns where each unknown ranges over a finite domain. We prove that such a system has a solution if and only if every finite subsystem has a solution. Moreover, we introduce the notion of an expanding system of equations and its partial solution and we give a necessary and sufficient condition for the existence of a partial solution of such a system of equations. Furthermore, we prove that Rado's theorem [1] is equivalent to the statement that if each equation of an expanding system of equations has a solution then the system has a partial solution.

In what follows we consider infinitely many (not necessarily denumerably many) unknowns (variables) $x_1, x_2, \ldots, x_j, \ldots$ ranging respectively over nonempty finite domains $D_1, D_2, \ldots, D_j, \ldots$. Moreover, by a function we mean a function of finitely many unknowns (variables). Hence, a function in the unknowns $x_i, \ldots, x_k$ is a mapping from $D_i \times \ldots \times D_k$. We do not impose any restriction (except for being nonempty) on the range of a function since that is not needed for our purpose.

From a given function we construct equations in the usual way. Thus, if

(1) $F_i(\ldots, x_j, \ldots)$

is a function then the configuration

(2) $F_i(\ldots, x_j, \ldots) = c_i$

is an equation, where c_i is an element of the range of the function given in (1). The notion of a solution of an equation as well as that of a system of equations is self-explanatory.

In the sequel, we let V denote a nonempty index set for the unknowns and we consider equations indexed by a nonempty set E. Although we make no restrictions (except for being nonzero) on the cardinalities of sets V and E, we would like to emphasize that each equation has finitely many unknowns and each unknown ranges over a nonempty finite domain.

Motivated by notation (2), we prove the following theorem.

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