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A NOTE ON IMPLICATIVE MODELS

E. L. MARSDEN

1. *Introduction.* Implicative models were first considered by Leon Henkin who explored the relation between certain formal (logical) systems and certain algebraic structures. More precisely, implicative models correspond to a logical system whose only logical connective is implication and whose laws are satisfied by classical, intuitionistic and modal logics.

Several examples of implicative models are Boolean lattices, Brouwerian semi-lattices, and closures algebras. Henkin's definition of an implicative model has been dualized to conform with common notation for Brouwerian semi-lattices. In this note it is shown that several significant results for Brouwerian semi-lattices also obtain in the setting of implicative models.

2. *Implicative Models*. An implicative model [2] is an algebraic system $\langle X, *, 1\rangle$ where X is a set, 1 is an element of X, and $*$ is a binary operation satisfying the axioms listed below. It is convenient to use the relation $\leq$ defined by $x \leq y$ if $x * y = 1$. The following axioms hold for all x, y, z in X:

A_1 $\quad y \leq x * y$
A_2 $\quad x * (y * z) \leq (x * y) * (x * z)$
A_3 $\quad x \leq 1$
A_4 $\quad x = y$ *when* $x \leq y$ *and* $y \leq x$.

Proposition 1.

(i) *If* $1 \leq x$, *then* $1 = x$.
(ii) $x * 1 = 1$.
(iii) $x * x = 1$.
(iv) $1 * x = x$.
(v) *If* $x \leq y$ *and* $y \leq z$, *then* $x \leq z$.
(vi) $\langle X, \leq\rangle$ *is a partially ordered set.*

Proposition 2.

(i) *If* $x \leq y$, *then* $y * z \leq x * z$.
(ii) $x * (y * z) = y * (x * z)$.

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