Notre Dame Journal of Formal Logic
Volume XIV, Number 1, January 1973
NDJFAM

FORMATION SEQUENCES FOR PROPOSITIONAL FORMULAS

MARTIN M. ZUCKERMAN

Formation sequences play a central role in Smullyan's elegant development of the propositional calculus, given in [1] and [2]. In the following, we modify the treatment of [1] in that we take only $\sim$ and $\vee$ as our undefined logical connectives; the definitions given in [1] are altered accordingly.*

Let $\mathcal{P}_0$ be a denumerable collection of symbols, called *propositional variables*. Let the four symbols

$$\sim, \vee, (,)$$

be distinct from each other and from the propositional variables. A *formation sequence* is defined, recursively, to be a finite sequence each of whose terms is either

(i) a propositional variable,
(ii) of the form $\sim P$, where P is an earlier term of the sequence, or
(iii) of the form $(P \vee Q)$, where P and Q are earlier terms of the sequence.

P is called a *formula* if there is a formation sequence, $\langle P_0, P_1, \ldots, P_N\rangle$ in which $P_N = P$; $\langle P_0, P_1, \ldots, P_N\rangle$ is then called a *formation sequence for P*. It follows directly from this definition that if $\langle P_0, P_1, \ldots, P_N\rangle$ is a formation sequence, then for each $K \leq N$, $\langle P_0, P_1, \ldots, P_K\rangle$ is a formation sequence for P_K. As the name suggests, formation sequences yield information concerning the manner in which formulas are constructed from propositional variables by means of connectives. Clearly, a formation sequence for a formula, P, is not unique.

Formulas of type (ii) are called *negations*; those of type (iii) are called *disjunctions*. It is well-known that for every formula P, exactly one of the following holds:

*This research was supported by a City University of New York Faculty Research Award, 1970.

Received August 1, 1970