Notre Dame Journal of Formal Logic
Volume XIV, Number 1, January 1973
NDJFAM

ON SOME MODELS OF MODAL LOGICS

R. J. BAXTER

The purpose of this note is to show that the models of the modal logics M, S4, *Brouwersche* and S5 defined by Drake in [1], following McKinsey [3], can be presented as models of the type defined by Kripke in [2].

Models based on a Boolean algebra $\mathfrak{A}$ were defined in [1] as follows. A model is a triple $\langle \mathfrak{A}, D, S \rangle$ where $\mathfrak{A}$ is a Boolean algebra, D is a maximal additive ideal of $\mathfrak{A}$, and S is a set of operators defined on elements of $\mathfrak{A}$ satisfying

a1) $\mathrm{s}(a \cup b) = \mathrm{s}(a) \cup \mathrm{s}(b)$,
a2) $\mathrm{s}(-a) = -\mathrm{s}(a)$ ($-a$ is the complement of a)

and

a3) *there is an* $\mathrm{s}_0 \in S$ *such that* $\mathrm{s}_0(a) = a$ *for all* $a \in A$.

In addition S may be assumed to satisfy one or both of

a4) *for each* $\mathrm{s}, \mathrm{s}' \in S$, *there is an* $\mathrm{s}'' \in S$ *such that* $\mathrm{s}\{\mathrm{s}'(a)\} = \mathrm{s}''(a)$ *for all* $a \in A$,
a5) *for any* $a_1, \ldots, a_n \in A$ *and* $\mathrm{s} \in S$, *there is an* $\mathrm{s}' \in S$ *such that* $\mathrm{s}\{\mathrm{s}'(a_1)\} = a_1, \ldots, \mathrm{s}\{\mathrm{s}'(a_n)\} = a_n$.

Defining an operation

$$*a = \bigcup_{\mathrm{s} \in S} \mathrm{s}(a),$$

corresponding to the modal operation of possibility, Drake showed that, if S is assumed to satisfy a1)-a3) {a1)-a4), a1)-a5), resp.}, then the triples $\langle \mathfrak{A}, D, S \rangle$ are characteristic for M(S4, S5). It is easy to show by the methods of [1] that, if S satisfies a1)-a3) and a5) then $\langle \mathfrak{A}, D, S \rangle$ is characteristic for the *Brouwersche* system.

In [2], triples $\langle G, K, R \rangle$ are defined with $G \in K$ and K is to be interpreted as a set of possible worlds. R is a relation on K and these triples are called model structures. A model structure is an M- (S4-, *Brouwersche*-, S5-, resp.) model structure if R is reflexive (reflexive and transitive, reflexive and symmetric, an equivalence relation). A model is a function $\Phi(A, H)$ where A ranges over subformulae of the given formula and H ranges

Received July 26, 1971