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ON A PROPERTY OF CERTAIN PROPOSITIONAL FORMULAE

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In [1] section **4** Łukasiewicz gives a theorem concerning the law of syllogism. The present paper presents a much more general theorem from which the Łukasiewicz theorem can be derived. Sections **1** and **2** present our theorem; a brief discussion of its application and its relationship to the Łukasiewicz theorem is given in section **3**.

1. *Preliminaries and Statement of Theorem.* We use 'P', 'Q', 'R', with and without subscripts to denote well-formed propositional formulae. '$\{P_1, \ldots P_n\}$', '$\{Q_1, \ldots Q_m\}$' and so on denote ordered sets of such formulae. 'Φ' is used for a constant operation under the substitution rule; 'Φ^n' denotes n repetitions of the operation; '$\Phi\{P_1, \ldots P_n\}$' is an abbreviation for '$\{\Phi P_1, \ldots \Phi P_n\}$'. '$\cup$' and '$\subset$' have their usual meanings. We use '$\sim$' to denote a relationship between an ordered set of propositional formulae and a single formula which is defined as follows.

Definition $\{P_1, \ldots P_n\} \sim Q$ is defined inductively in two steps:
a. Let Q be a member of $\{P_1, \ldots P_n\}$: then $\{P_1, \ldots P_n\} \sim Q$.
b. For some R, let $\{P_1, \ldots P_n\} \sim R$ and let $\{P_1, \ldots P_n\} \sim CRQ$: then $\{P_1, \ldots P_n\} \sim Q$.

Less formally, our relationship holds between a formula and any ordered set of formulae of which it is a member, or from which it can be obtained by one or more applications of Modus Ponens. Our theorem can now be stated.

Theorem *For any well-formed formula of the form* $CP_1 \ldots CP_nCQ_1 \ldots CQ_{m-1}Q_m$ $(m, n \geq 1)$ *if the following three conditions are satisfied:*

a. $\Phi CQ_1 \ldots CQ_{m-1}Q_m = CP_1 \ldots CP_nCQ_1 \ldots CQ_{m-1}Q_m$
b. Q_m *is elementary*
c. $\{P_1, \ldots P_n, Q_1, \ldots Q_{m-1}\} \sim Q_m$

then

$$\Phi\{P_1, \ldots P_n\} \cup \Phi^2\{P_1, \ldots P_n\} \cup \ldots \Phi^m\{P_1, \ldots P_n\} \sim CP_1 \ldots CP_nCQ_1 \ldots CQ_{m-1} Q_m$$

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