Notre Dame Journal of Formal Logic
Volume XIV, Number 1, January 1973
NDJFAM

A NOTE ON A THEOREM OF C. YATES

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1. *Introduction.* Let E denote the collection of all non-negative integers. We recall from [2] that a one-to-one function t_n (from E into E) is *regressive* if the mapping $t_{n+1} \to t_n$ has a partial recursive extension; and is *retraceable* if it is both strictly increasing and regressive. An infinite set is said to be *regressive* if it is the range of a regressive function; and is *retraceable* if it is the range of a retraceable function. A one-to-one function a_n is *indexed* if the mapping $a_n \to n$ has a partial recursive extension; and a set is *indexed* if it is the range of an indexed function. In [4] C. Yates proved the following result:

Theorem A. (Yates). *Let α be an infinite set. Then α is strongly hyper-hyperimmune $\Longleftrightarrow \alpha$ contains no infinite retraceable subset.*

In this paper we arrive at a new proof of this result. It is somewhat easier than the proof in [4] (see also: [3, pp. 250-251]), and also, it makes use of a basic property of indexed sets.

2. *Indexed sets.* Let $\{w_n\}$ denote the usual effective enumeration of the collection of all recursively enumerable sets. We call a sequence $\{w_{f(x)}\}$ an *array* if

(a) f is a one-to-one recursive function,
(b) for each x, $w_{f(x)} \neq \phi$, and
(c) for each x and y, if $x \neq y$ then $w_{f(x)} \cap w_{f(y)} = \phi$.

We recall from [3, p. 250] that an infinite set α is said to be *strongly hyperhyperimmune* if for every array $\{w_{f(x)}\}$, there is a number x such that $w_{f(x)} \cap \alpha = \phi$.

Theorem 1. *Let α be an infinite set. Then α is a strongly hyperhyperimmune $\Longleftrightarrow \alpha$ contains no infinite indexed subset.*

Proof. ($\Longrightarrow$) Assume that α is strongly hyperhyperimmune and suppose that α contains an infinite indexed subset. Let a_n be an indexed function that ranges over a subset of α and let p denote a partial recursive function such that, for each number n,

Received March 13, 1971