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TOLERANCE GEOMETRY

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1. *Introduction** In his paper on visual perception [7], Zeeman points out that any model of perception must take account of the fact that we cannot distinguish between points that are sufficiently close. A similar observation has been made for choice behavior by Luce [1]. Zeeman's observation leads him directly to a notion of a "tolerance" within which "we allow an object to move before we notice any difference." Other authors use the terms "threshold" and "just noticeable difference," for the same notion.

Zeeman defines a *tolerance* I on a set A as a binary relation on A which is reflexive and symmetric, and he calls the pair (A, I) a *tolerance space*. We shall use the more common term *graph* for this concept and prefer to think of tolerance spaces or tolerance relations as graphs with more specialized properties, motivated by the notion of "closeness." Zeeman studies various properties of and relations between tolerance spaces (graphs), using topological techniques.

In studying visual perception, it is convenient to distinguish between physical space and (subjective) visual space, the space from which we draw our "conscious" perceptions. It has been observed in the literature that visual space has a non-Euclidean geometry (see Roberts and Suppes [4]). To determine what this geometry is, observed relations such as betweenness, alignment, perpendicularity (of two aligned sets of points), parallelism, etc. are studied, and their properties are determined.

In order to study visual geometry, to take account of the tolerance effect, it seems desirable to replace classical primitives, such as betweenness, straightness, perpendicularity, and parallelism, with more general notions, obtained from the classical ones by substituting closeness for identity. We shall use the term *tolerance geometry* for any geometry whose

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