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Notre Dame Journal of Formal Logic
Volume XIII, Number 4, October 1972
NDJFAM

SEMI-INTUITIONISTIC SET THEORY

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The Zermelo-Fraenkel axioms represent set theory as a kind of "constructive" enterprise involving the production of new sets from old ones by means of carefully delineated processes. This could suggest that for the purposes of formalization the usual classical first-order logic may not be altogether appropriate (see [3]). In any case it seems worthwhile to investigate what happens when this quasi-constructive attitude is reflected in the very logic of the system, and what follows is an attempt to formulate a definite formal system along such lines and to compare it with the usual formulation of Zermelo-Fraenkel set theory as a classical first-order theory.*

Our system will be "constructive" in the sense that the law of the excluded middle will be postulated only for predicates with quantifiers restricted to certain terms representing sets. To get a sufficiency of such terms, we shall introduce various term-operators corresponding to the constructions postulated in the axioms. If "ZF_c" represents the usual classical formulation of Zermelo-Fraenkel set theory (including the axioms of regularity and choice), we may represent our system by "ZF_s*", where the "s" represents the "semi-intuitionistic" logic and the asterisk represents the addition of the term-operators. In section 1 we shall present the system ZF_s*, in 2 discuss the development of set theory within ZF_s*, and in 3 add some further remarks.

1. *The System* ZF_s*. The symbols are the constants $0, \omega$, the variables $x_0, x_1, x_2, \ldots$, the operators U, $\mathcal{P}$, $\mathcal{C}$, the predicates $=, \epsilon$, the connectives $\rightarrow, \&, \vee, -, \forall, \exists$, and the punctuational symbols $(,), [,], \{, \}, :$, and $,$. In the metalanguage we shall represent variables by lower-case letters a, b, $c, \ldots$, terms by early capitals A, B, $C, \ldots$, and formulas by later

*Most of this work was done in connection with a National Science Foundation program of Research Participation for College Teachers conducted at the University of Oklahoma in the summer of 1968.

Received September 6, 1970