Notre Dame Journal of Formal Logic
Volume XIII, Number 4, October 1972
NDJFAM

SOLUTION TO THE PROBLEM CONCERNING THE BOOLEAN BASES FOR CYLINDRIC ALGEBRAS

BOLESŁAW SOBOCIŃSKI

In [1],[1] p. 162, Definition 1.1.1, a cylindric algebra of dimension α is defined, as follows:

(**A**) *A cylindric algebra of dimension* α, *where* α *is any ordinal number, is an algebraic structure*

$$\mathfrak{A} = \langle A, +, \times, -, 0, 1, \mathbf{c}_\kappa, \mathbf{d}_{\kappa\lambda}\rangle_{\kappa,\lambda<\alpha}$$

such that 0, 1, *and* $\mathbf{d}_{\kappa\lambda}$, *are distinguished* (constant)[2] *elements of the carrier set* A (*for all* $\kappa, \lambda < \alpha$), - *and* $\mathbf{c}_\kappa$ *are unary operations on* A (*for all* $\kappa < \alpha$), + *and* $\times$ *are binary operations on* A, *and such that the following postulates are satisfied for any* $x, y \in A$ *and any* $\kappa, \lambda, \mu < \alpha$:

C0 The structure $\langle A, +, \times, -, 0, 1\rangle$ is a **BA**;
C1 $[\kappa]: \kappa < \alpha . \supset . \mathbf{c}_\kappa 0 = 0$;
C2 $[x\kappa]: x \in A . \kappa < \alpha . \supset . x \leq \mathbf{c}_\kappa x$ (*i.e.*, $x + \mathbf{c}_\kappa x = \mathbf{c}_\kappa x$);
C3 $[xy\kappa]: x, y \in A . \kappa < \alpha . \supset . \mathbf{c}_\kappa(x \times \mathbf{c}_\kappa y) = \mathbf{c}_\kappa x \times \mathbf{c}_\kappa y$;
C4 $[x\kappa\lambda]: x \in A . \kappa, \lambda < \alpha . \supset . \mathbf{c}_\kappa \mathbf{c}_\lambda x = \mathbf{c}_\lambda \mathbf{c}_\kappa x$;
C5 $[\kappa]: \kappa < \alpha . \supset . \mathbf{d}_{\kappa\kappa} = 1$;
C6 $[\kappa\lambda\mu]: \kappa, \lambda, \mu < \alpha, \kappa \neq \lambda, \mu . \supset . \mathbf{d}_{\lambda\mu} = \mathbf{c}_\kappa(\mathbf{d}_{\lambda\kappa} \times \mathbf{d}_{\kappa\mu})$;
C7 $[x\kappa\lambda]: x \in A . \kappa, \lambda < \alpha . \kappa \neq \lambda . \supset . \mathbf{c}_\kappa(\mathbf{d}_{\kappa\lambda} \times x) \times \mathbf{c}_\kappa(\mathbf{d}_{\kappa\lambda} \times -x) = 0$.

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1. An elementary familiarity with the theory of cylindric algebras and an acquaintance with the papers [2], [3] and [4] is presupposed. Concerning the symbols used in this paper it should be remarked that instead of "$a \cdot b$" which is used in [1] I am using "$a \times b$", and that instead of "$\overline{a}$" used in [3] I am using here "$-a$". An enumeration of the algebraic tables, *cf.* section **3** below, is a continuation of the enumeration of such tables given in [3], [5] and [6].

2. In this paper the difference between the distinguished elements and the constant elements will be disregarded because it is unessential for our present research.

Received November 1, 1971