Notre Dame Journal of Formal Logic
Volume XIII, Number 3, July 1972
NDJFAM

ε-CALCULUS BASED AXIOM SYSTEMS FOR SOME PROPOSITIONAL MODAL LOGICS

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1 *Introduction*: Instead of considering propositional S4 (say) as a theory built on top of classical *propositional* logic, we can consider it as the classical *first order* theory of its Kripke models. This first order theory can be formulated in any of the ways first order theories usually are: tableau, Gentzen system, natural deduction, conventional axiom system, ε-calculus. If the first order theory of Kripke S4 models can be given in a formulation which is technically easy to use, the result is a convenient S4 proof system. Thus, in [2] we gave a tableau formulation of the S4 model theory which dealt automatically with the peculiarities of Kripke models, and produced a proof system for S4 (and similarly for other modal logics) which is simple to apply. In this paper we give what is essentially an ε-calculus formulation of the Kripke model theory of S4 (and S5, T, B, and also first order versions) which also automatically treats the peculiarities of the Kripke models. It is less convenient in use than the tableau system of [2] but still has a curious intrinsic interest.

2 *Propositional systems*: We begin with S4, which we formulate as a classical theory in which atomic formulas, as defined below, are strings of symbols which include modal formulas as substrings. We take $\sim$, $\wedge$, and $\diamond$ as primitive. Modal formulas are defined as usual. We use $P_1, P_2, \ldots,$ $Q_1, Q_2, \ldots$ to stand for modal formulas beginning with an occurrence of $\diamond$, and X and Y to stand for arbitrary modal formulas. Let * be some new symbol. By an *atomic formula* (of the classical theory we are constructing) we mean a string of symbols of the form $*P_1, \ldots, P_n*X$ (we adopt the convention that n may be 0, i.e. $**X$ is an atomic formula). *Formulas* are built up in the usual way from atomic formulas using $\sim$ and $\wedge$ as primitive.

We have the single rule of modus ponens, and as axioms we take all classical tautologies, and all formulas of the form (where again n may be 0):

(1) $*P_1, \ldots, P_n*X \wedge Y \equiv *P_1, \ldots, P_n*X \wedge *P_1, \ldots, P_n*Y$
(2) $*P_1, \ldots, P_n*\sim X \equiv \sim *P_1, \ldots, P_n*X$

Received April 22, 1970