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AXIOMS FOR GENERALIZED NEWMAN ALGEBRAS

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Introduction In [2], Newman exhibited a remarkable set of axioms defining an algebra which characterizes the direct join of a Boolean algebra and a non-associative Boolean ring with unit. Subsequently this algebra has been named Newman algebra and, from the point of view of axiomatics, has been intensively studied (see references). In a later paper [3], Newman exhibited an independent set of axioms characterizing the direct join of a generalized Boolean algebra and a Boolean ring. Newman's exposition is lengthy and the absence of postulated commutative and associative laws (even though at great length provable) necessitates intricate computation. The purpose of this note is to give a set of independent axioms for this generalized Newman algebra containing two fewer axioms than Newman's system. The new system, even though it contains fewer axioms than the original system, can be used to give a more economical proof of Newman's aforementioned representation theorem.

The axiom systems Following Newman [3], we define a *generalized Newman algebra* to be an algebra $\langle A; +, \cdot\rangle$ with two binary operations (called addition and multiplication, respectively) satisfying the axioms

[N1] $a(b + c) = ab + ac$
[N2] $(a + b)c = ac + bc$
[N3] $aa = a$
[N4] $a(bb) = (ab)b$
[N5] *there exists* $\omega_1 \in A$ *such that the equations* $x + a = b$, $xa = \omega_1$ *have a solution whenever* $ab = a$.
[N6] *there exists* $\omega_2 \in A$ *such that the equations* $x + a = b$, $ax = \omega_2$ *have a solution whenever* $ba = a$.

In Newman's terminology the distinguished elements ω_1, ω_2 are called the *left, right omegas* of A, respectively. A solution of the simultaneous equations in [N5] is called a *b-complement of a*. If $a, m \in A$ satisfy the relation $am = a$ ($ma = a$) then m is called a *right* (*left*) *majorant of a.* Under the additional assumption that multiplication is commutative we drop the

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