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AN ADDITIONAL REMARK ON SELF-CONJUGATE FUNCTIONS ON BOOLEAN ALGEBRAS

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In this note I add several remarks to my paper [1]. Let f be a self-conjugate function on a Boolean algebra, in [1] it was shown that if $f^n = \mathrm{id}$ and $f \neq \mathrm{id}$, then n is even and $f^2 = \mathrm{id}$. Necessary and sufficient conditions for a self-conjugate function f to have the property $f^2 = f$ were given. Here we extend results of this type. The references in the proofs are from [1]. Let

$$\mathfrak{A} = \langle A, +, \cdot, -, 0, 1\rangle$$

be a **BA**.

Lemma *If f: $A \to A$ is self-conjugate, then*

$$x \leq f^2(x) \leq f^4(x) \leq \ldots \leq f^{2n}(x) \leq \ldots$$

and

$$f(x) \leq f^3(x) \leq f^5(x) \leq \ldots \leq f^{2n+1}(x) \leq \ldots$$

for any $x \in A$, and all $n > 0$.

Proof: By replacing x by $f(1)$ and y by $f^n(x)$ in 1.2(iii) we obtain

$$f^n(x) = f(1) \cdot f^n(x) \leq f(1 \cdot f^{n+1}(x)) = f^{n+2}(x)$$

for any $n > 0$. Now if we show that $x \leq f^2(x)$ we are done. Let $c = x - f^2(x)$. By 1.1, 1.3, and 1.5,

$$f^2(x) \cdot c = 0 \leftrightarrow f(x) \cdot f(c) = 0 \leftrightarrow f(c) = 0 \leftrightarrow c = 0.$$

Hence $x \leq f^2(x)$ as desired.

Theorem *If f: $A \to A$ is self-conjugate, $f^n = f^m$, $n < m$ and for $i \leq n$, $j < m$, $f^i \neq f^j$, then*

(i) *$m = n + 2$, if n, m have the same parity,*
(ii) *$m = n + 1$, if n, m have different parities.*

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