

## THE ELIMINABILITY OF THE ACTUALITY OPERATOR IN PROPOSITIONAL MODAL LOGIC

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The modal logic S5 is determined by the class of Kripke models in which the accessibility relation is the universal relation over the set of possible worlds or is an equivalence, or by the class of "pure" Kripke models, with no epicycles or accessibility relation,  $Lp(Mp)$  being counted true at a world in a model structure just in case  $p$  is true at every (at least one) world of that structure. It has been argued (*cf.* [4], in which ideas related to the actuality operator are also introduced, especially page 475) that it is the correct modal logic for the notions of metaphysical or logical necessity and possibility.

S5 may be given a natural deduction formulation in the manner of Fitch (*cf.* [1]) by adding to a natural deduction formulation of the classical propositional calculus (e.g., that of [1] or of [5]) a special kind of subordinate proof, called a *strict* subproof, into which formulas whose initial symbols are modal operators (we assume a notation like that of Łukasiewicz, in which the initial symbol of a complex formula is always its main connective) may be reiterated without change and other formulas not at all, and the following rules:

(*L*-introduction):  $Lp$  is a direct consequence (**d.c.**) of a strict subordinate proof with no hypothesis whose last item is  $p$ .

(*L*-elimination):  $p$  is a **d.c.** of  $Lp$ .

(*M*-introduction):  $Mp$  is a **d.c.** of  $p$ .

(*M*-elimination):  $Mq$  is a **d.c.** of  $Mp$  and a strict subordinate proof whose only hypothesis is  $p$  and whose last item is  $q$ .

(Negative modality rules):  $NLp$ ,  $MNp$ ,  $NMp$ , and  $LNp$  are **d.c.**'s respectively of  $Mnp$ ,  $NLp$ ,  $LNp$ , and  $NMp$ .

These rules are readily seen to be equivalent to a set presented in the first appendix to [3], where it is shown that they are at least as strong as an axiomatic formulation proved elsewhere in [3] to be complete relative to the Kripke semantics for S5. That the natural deduction system is sound

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