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# THE ONTOLOGICAL THEOREM

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Charles Hartshorne presented a proof of the Ontological Argument containing the postulate, $p \rightarrow \sim\Diamond\sim p$, which he calls "Anselm's Principle."[1] Few of us, however, would accept even the weaker expression, $p \supset \sim\Diamond\sim p$, as plausible, and most of us would reject it on intuitive grounds.[2] If we do deny it, then strange things happen in any standard modal system that is sufficiently complex (*e.g.*, Feys' System T) to contain an equivalent of the rule:

**(R)** $\vdash \alpha \therefore \vdash \Diamond\beta \supset \Diamond(\alpha\cdot\beta)$.

For example:

| | | |
|---|---|---|
| (1) | $\sim(p \supset \sim\Diamond\sim p)$ | |
| (2) | $\vdash p\cdot\Diamond\sim p$ | 1, **PC** |
| (3) | $\vdash p$ | 2, **PC** |
| (4) | $\vdash \Diamond \sim p \supset \Diamond(p\cdot\sim p)$ | 3, **R** |
| (5) | $\vdash \Diamond \sim p$ | 2, **PC** |
| (6) | $\vdash \Diamond(p\cdot\sim p)$ | 4,5, **PC** |
| (7) | $\vdash \sim(p \supset \sim\Diamond\sim p) \supset \Diamond(p\cdot\sim p)$ | 1-6, **PC** |
| (8) | $\vdash \sim\Diamond(p\cdot\sim p) \supset (p \supset \sim\Diamond\sim p)$ | 7, **PC** |

Let us call (8) the Ontological Theorem. If we assume the law of Non-Contradiction then we are forced by the Ontological Theorem to accept

1. Charles Hartshorne, *The Logic of Perfection* (La Salle: Open Court Publishing Company, 1962), pp. 49ff, esp. p. 51.

2. Nicholas Rescher, in "On the formalization of two modal theses," *Notre Dame Journal of Formal Logic*, vol. II (1961), pp. 154-157, formulates "No 'mere fact' or 'merely contingent proposition' entails a necessary proposition" as a tautology by imbedding the consequent ($\sim\Box q$) as a conjunctive component ($\Diamond\sim q$) of the antecedent of the strict implication. His formulation does avoid the *consequentia mirabilis*, which was his purpose in having $\Diamond\sim q$ in the antecedent; but the Ontological Theorem in no way involves the *consequentia mirabilis*, either.

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