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MODEL THEORETICAL INVESTIGATION OF THEOREM PROVING METHODS

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1 *Introduction* From the literature of the logical deduction theory, several different methods are known for theorem proving in different calculi. Usually, these methods are purely syntactically founded, which, in our opinion, often leads to the mystification of syntax. In the present paper we have tried to discuss these methods from a model-theoretical point of view. The basic facts serving as foundation for our treatment are also discussed.

1.1 *The general concept of language* A language is represented by a triple $\mathcal{L} = \langle \mathsf{F}, \mathbf{M}, \models \rangle$, where F is the syntax of the language, i.e., a certain set of words in a denumerably infinite alphabet and $\langle \mathbf{M}, \models \rangle$ is the semantics with $\mathbf{M}$ being the class of models and $\models$ the validity relation ($\models \subseteq \mathbf{M} \times \mathsf{F}$). Let X be a finite alphabet and X^* the set of all possible words described by the alphabet X. Then the syntax F of the language $\mathcal{L} = \langle \mathsf{F}, \mathbf{M}, \models \rangle$ is usually given as follows:

(1) Some elements of X^* are defined as elements of F;
(2) New elements of F are constructed from the existing ones by using a number of generator rules;
(3) F contains no further element different from those obtained according to (1) and (2).

If $\varphi \in \mathsf{F}$ and $\mathfrak{A} \in \mathbf{M}$, then $\mathfrak{A} \models \varphi$ denotes that φ is valid in $\mathfrak{A}$. In other words, $\mathfrak{A} \models \varphi$ denotes that $\mathfrak{A}$ is a model of φ. If the case $\mathfrak{A} \models \varphi$ is not satisfied, this will symbolically be denoted by $\mathfrak{A} \not\models \varphi$.

1.2 *Basic notations* The notations $\Rightarrow$ and $\Leftrightarrow$ stand for "it follows" and "if and only if", respectively. It is to be noted that, in proofs, the notations $\Rightarrow$ and $\Leftarrow$ will also be used to indicate in which direction a statement containing $\Leftrightarrow$ is being proved. The letter "d" above the notations $=$ or $\Leftrightarrow$ is used to indicate that a new concept is defined. Functions will sometimes be considered as sets of pairs. $\mathsf{Rg}f$ and $\mathsf{Do}f$ denote the range and the domain of a function f, respectively. $\asymp$ denotes the graphical equality, ω denotes the first transfinite ordinal. $|A|$ denotes the cardinal number of the

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