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SOME OBSERVATIONS ON A METHOD OF McKINSEY

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In an early paper[1] J. C. C. McKinsey proved that no one of the intuitionist connectives $\neg$, $\wedge$, $\vee$, and $\supset$ is definable in terms of the remaining three. Those who are familiar with the details of his argument can have little doubt concerning its soundness. Nonetheless, McKinsey's characterization of that argument is certainly defective.

> More generally, we can see, that if three of the operations we are considering are class-closing on some proper sub-class of the elements of a matrix, while the fourth is not class-closing on this proper subclass, then the fourth operation is not definable in terms of the other three.[2]

Were this account correct, it would be all too easy to show that no connective from a propositional system is definable in terms of other connectives from that system.

Proof: Let $\oplus$ be any n-place connective from an arbitrary propositional system Σ, and let Δ be any set of other connectives from Σ. Consider now the matrix $\mathfrak{N}$. The elements of $\mathfrak{N}$, both of which are designated, are 0 and 1. The operation that $\mathfrak{N}$ associates with $\oplus$ is that n-ary operation on $\{0, 1\}$ that always assumes the value 0. The operations that $\mathfrak{N}$ associates with the members of Δ are operations that always assume the value 1. It is clear that $\mathfrak{N}$ is a matrix *for* Σ (i.e. that each thesis of Σ is a tautology under $\mathfrak{N}$), that the operations $\mathfrak{N}$ assigns to the members of Δ are class closing on $\{1\}$, and that the operation $\mathfrak{N}$ assigns to $\oplus$ is not class closing on $\{1\}$. Thus, if McKinsey's account were correct, $\oplus$ would not be definable in terms of the members of Δ.

A more interesting matrix in this respect, but one that assumes that the rule of substitution holds for Σ, is the Lindenbaum matrix for Σ. A

1. McKinsey, J. C. C., "Proof of the Independence of the Primitive Symbols of Heyting's Calculus of Propositions," *The Journal of Symbolic Logic*, vol. 4 (1939), pp. 155-158.

2. *Ibid.*, p. 156.

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