Notre Dame Journal of Formal Logic
Volume XIX, Number 2, April 1978
NDJFAM

# A NOTE CONCERNING THE NOTION OF MEREOLOGICAL CLASS

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**1** In mereology we have a number of equivalences which in various ways characterize the notion of mereological class. Some of these equivalences have been used, in some systems of mereology, as definitions while others have been proved in these systems as theorems. In the present note I shall be concerned with the following three equivalences:

*E1* $[Aa] \therefore A \varepsilon A : [B] : B \varepsilon a \mathbin{.\supset.} B \varepsilon \mathbf{el}(A) : [B] : B \varepsilon \mathbf{el}(A) \mathbin{.\supset.} [\exists CD] . C \varepsilon a . D \varepsilon \mathbf{el}(B) . D \varepsilon \mathbf{el}(C) \mathbin{:\equiv.} A \varepsilon \mathbf{Kl}(a)$

*E2* $[Aa] \therefore A \varepsilon A : [B] : [\exists C] . C \varepsilon \mathbf{el}(A) . C \varepsilon \mathbf{el}(B) \mathbin{.\equiv.} [\exists DE] . D \varepsilon a . E \varepsilon \mathbf{el}(B) . E \varepsilon \mathbf{el}(D) \mathbin{:\equiv.} A \varepsilon \mathbf{Kl}(a)$

*E3* $[Aa] :: A \varepsilon A \therefore [B] \therefore A \varepsilon \mathbf{el}(B) \mathbin{.\equiv:} [C] : C \varepsilon a \mathbin{.\supset.} C \varepsilon \mathbf{el}(B) \mathbin{\therefore\equiv.} A \varepsilon \mathbf{Kl}(a)$

Equivalence *E1*, which is due to Leśniewski, is normally used as a definition in systems of mereology in which the notion of mereological element serves as the only undefined mereological notion.[1] Thus, for instance, *E1* is used as a definition in the system based on the following single axiom:

*AA1* $[AB] :: A \varepsilon \mathbf{el}(B) \mathbin{.\equiv\because} B \varepsilon B \because [Ca] \because [D] \therefore D \varepsilon C \mathbin{.\equiv:} [E] : E \varepsilon a \mathbin{.\supset.} E \varepsilon \mathbf{el}(D) : [E] : E \varepsilon \mathbf{el}(D) \mathbin{.\supset.} [\exists FG] . F \varepsilon a . G \varepsilon \mathbf{el}(E) . G \varepsilon \mathbf{el}(F) :: B \varepsilon \mathbf{el}(B) . B \varepsilon a \mathbin{::\supset.} A \varepsilon \mathbf{el}(C)$[2]

It is not difficult to see that *E1* is, in a sense, embedded in *A1*, whose meaning becomes clearer once we have realized that the set of presuppositions consisting of *AA1* and *E1* is inferencially equivalent to the set of presuppositions consisting of *E1* and

*AA1.1* $[AB] \therefore A \varepsilon \mathbf{el}(B) \mathbin{.\equiv:} B \varepsilon B : [a] : B \varepsilon \mathbf{el}(B) . B \varepsilon a \mathbin{.\supset.} A \varepsilon \mathbf{el}(\mathbf{Kl}(a))$

With the aid of symbols we state this equivalence thus: $\{AA1, E1\} \rightleftarrows \{AA1.1, E1\}$, and we note that in $\{AA1, E1\}$ *E1* can be regarded as a definition whereas in $\{AA1.1, E1\}$ it cannot be so regarded in view of the fact that the notion of '**Kl**' already occurs in *AA1.1*. Consequently, $\{AA1.1, E1\}$ must be treated as an axiom system involving two undefined mereological notions, i.e., '**el**' and '**Kl**'.

In 1954 I noticed that *E2* could be used as the definition of '**Kl**' in a

*Received March 19, 1974*