

MODEL THEORY OF ALTERNATIVE RINGS

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Introduction Recently much work has been done in applying various techniques developed in logic to the study of associative rings [4, 6, 9, 18, 42]. As a result of this inquiry we have a better understanding of what certain general model theoretic properties mean in well-known mathematical contexts. In this paper*, although we continue this program of examining logic in the context of ring theory, we are concerned with a larger class of rings- alternative rings. The class of alternative rings is axiomatizable by the standard axioms of ring theory with the associative axiom replaced by the sentence:

$$\forall x \forall y ((xx)y = x(xy) \wedge (yx)x = y(xy)).$$

Note that an alternative ring may be associative. A very useful characterization of alternative rings which shows their relationship to associative rings is Artin's Theorem [29]:

A ring is alternative if and only if all of its subrings generated by two elements are associative.

The canonical examples of alternative rings are the Cayley-Dickson algebras. Section 1 contains a brief introduction to Cayley-Dickson algebras.

We begin the mathematics of this paper in section 2 with a model theoretic exploration of split Cayley-Dickson algebras. We first show that

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