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LOGICS FOR KNOWLEDGE, POSSIBILITY, AND EXISTENCE

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In [2] completeness proofs were set out for several possibility pre-supposition free logics. Use was made of the kind of semantics to be found in Hintikka's work, especially in [3] and in *Knowledge and Belief* [4]. It is of interest to extend the possibility pre-supposition free logics by means of epistemic modalities similar to those in *Knowledge and Belief*, and by means of alethic modalities. In what follows we will be concerned with extensions of $\mathbf{QH}^2_=$, or systems isomorphic with $\mathbf{QH}^2_=$. Such extended logics could deal with sentences such as "John knows that the round square is an impossible object," "Everybody knows that Mr. Pickwick is an imaginary character," and "Mr. Pickwick knows who the Queen is."

As in [9] we use the quantifiers π and Σ to range over objects said to be real and objects said to be possible. We will also use the quantifiers U and E, as in [4], to range over objects said to be real or existing. The formula '$(\Sigma x)(x = a)$' would be translated as 'a is a possible object', and '$(\mathsf{E}\, x)(x = a)$' as 'a exists'. In order to avoid some of the problems which arise in [4] as a result of reading '$K_a p$' as 'a knows that p' and reading '$P_a p$' as 'It is possible, for all that a knows, that p', and holding '$P_a p \equiv \sim K_a \sim p$' we have two epistemic operators, ^LP and K. '$K_a p$' is read as above. '$^LP_a p$' is read as '$P_a p$' above. Whereas it is indefensible in the logic in [3] to say '$\sim K_a \mathbf{T}$' where $\mathbf{T}$ is a tautology, in the logics set out below we can defensibly say '$\sim K_a \mathbf{T}$' even though it is clearly indefensible to say '$^LP_a \sim \mathbf{T}$'.

1 Primitive symbols:

improper symbols $\supset\ \sim\ \pi\ \cup\ K\ {}^LP\ (\ )\ \Diamond$
bound personal variables $x_0, y_0, z_0, x_1, y_1, z_1, x_2, y_2, z_2, \ldots$
free personal variables $a_0, b_0, c_0, a_1, b_1, c_1, a_2, b_2, c_2, \ldots$
bound impersonal variables $i_0, j_0, k_0, i_1, j_1, k_1, i_2, j_2, k_2, \ldots$
free impersonal variables $s_0, t_0, u_0, s_1, t_1, u_1, s_2, t_2, u_2, \ldots$
propositional variables $p_0, q_0, r_0, p_1, q_1, r_1, p_2, q_2, r_2, \ldots$
n-ary predicate variables $(n \geqslant 1)$ $F_0^n, G_0^n, H_0^n, F_1^n, G_1^n, H_1^n, \ldots$
predicate constants $=$, $\mathbf{E}$

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