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A NOTE ON THE DECOMPOSITION OF THEORIES WITH RESPECT TO AMALGAMATION, CONVEXITY, AND RELATED PROPERTIES

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1 *Introduction* If T is any theory, it is well known that $T_\forall$, the universal part of T, can be uniquely represented as the intersection of irreducible components S_i; and, corresponding to this representation, there is a decomposition of the class $\mathcal{M}(T)$ of models of T into subclasses $\mathcal{M}(T \cup S_i)$. [This decomposition generalizes, for example, the classification of fields according to their characteristic.] In this note it is made clear, first, under what conditions the classes $\mathcal{M}(T \cup S_i)$ are mutually disjoint. This result is then used to show that any theory T having the amalgamation property can be decomposed into theories T_i such that each T_i has the joint extension property as well as the amalgamation property, and the classes $\mathcal{M}(T_i)$ are mutually disjoint. Then, turning to convex theories, it is shown that there is a one-to-one correspondence between the core models of a convex theory T and the components of $T_\exists$, hence T can be decomposed (according to the components of $T_\exists$) into convex theories with a unique core model. Decomposition results with similar intent have been obtained by Fisher and Robinson in [1], and by Fisher, Simmons, and Wheeler in [2].

We assume, throughout, that $\mathcal{L}$ is a countable, finitary, first-order language. A *theory* T is a consistent set of sentences of $\mathcal{L}$; $\mathcal{M}(T)$ is the class of models of T and, if $\mathfrak{A}$ is a structure of $\mathcal{L}$, $\mathrm{Th}(\mathfrak{A})$ is the set of all the sentences which are true in $\mathfrak{A}$. $\forall_1$ will designate the set of universal formulas of $\mathcal{L}$, and $\exists_1$ the set of existential formulas. $T_\forall$ designates the universal part of a theory T, and $T_\exists$ the existential part of T. By an *irreducible ideal* of $\forall_1$ (respectively $\exists_1$), we mean a deductively closed set S of universal (respectively existential) sentences such that $\phi \vee \psi \in S$ implies $\phi \in S$ or $\psi \in S$. A *component* of $T_\forall$ (respectively $T_\exists$) is a minimal irreducible extension of $T_\forall$ (respectively $T_\exists$).

2 *Components and the conditional joint extension property* Let T be a theory, let P be a component of $T_\forall$, and let $*P = \{\varepsilon \in \exists_1 : \neg\varepsilon \notin P\}$. It is trivial to verify that $*P$ is an irreducible ideal of $\exists_1$, and that $T \cup *P$ is consistent. Furthermore, $*P$ is maximal (among the ideals of $\exists_1$) with respect to being

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