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CRITICAL POINTS OF NORMAL FUNCTIONS. II

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Unless the contrary is made explicit, the notation and terminology of this present note will follow that in [1]. Perhaps the main difference lies in our concept of function; we are now more restrictive, and adopt the convention that all functions mentioned have domain ON. In the results that we are about to present,* the number 0 has the annoying habit of appearing as a special case to be considered with a good deal of frequency. We cannot eradicate this entirely, but can expedite matters somewhat by admitting 0 to the domain of the cofinality function cf, with the definition "$\mathrm{cf}(0) = 0$" (we do *not*, however, admit 0 to the class of regular ordinals). Thus we have $\mathrm{cf}(\alpha) \leqslant 1$ if and only if $\alpha = 0$ or $\alpha = \beta + 1$ for some β. By the prime component representation of an ordinal $\alpha > 0$, we mean the unique representation $\alpha = \rho_0 + \rho_1 + \ldots + \rho_n$, where each ρ_m is a prime component, and $\rho_m \geqslant \rho_{m+1}$ for $m < n$.

Let X be a class. We shall often enumerate X as (x_ξ), where the subscripts range over some ordinal (if X is a set) or over ON (if X is a proper class); in each case the subscript range will be clear from the context. Whenever such an enumeration is given, it will be assumed to be increasing.

Definition 1 A proper class $X = (x_\xi)$ is called "appropriate" if the following conditions are satisfied.

(1) If $x_0 \neq 0$, then $x_0 = \omega^\gamma$ for some γ such that $\mathrm{cf}(\omega^\gamma) = \omega$:
(2) For each ξ, $x_{\xi+1} - x_\xi = \omega^\gamma$ for some γ such that $\mathrm{cf}(\omega^\gamma) \leqslant \omega$.
(3) For each $\lambda \in \mathrm{LIM}^*$, $x_\lambda = \lim_{\xi<\lambda} x_\xi$.

We wish to show that for any class X, $X = \mathbf{CR}_f$ for some normal function f if and only if X is appropriate.

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