MODAL TREES FOR T AND S5

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1 The simplest decision procedure for the classical sentential calculus is the tree method given by R. Jeffrey in [1]. In this paper we describe an extension of the tree method to give a correspondingly simple decision procedure for the sentential modal logics T and S5. Familiarity with [1] is assumed. We aim just to give enough detail for someone familiar with the method and terminology of [1] to carry out tests for validity and consistency in T and S5. The method described can be adapted to provide decision procedures for B and S4. The rules for these systems are more complex than those for T and S5 and will not be explicitly dealt with here.

2 *Syntax* This is standard except (i) all sentence letters are given superscripts 0, 1, 2, . . .; and (ii) an expression is a wff if and only if it *both* satisfies the usual recursive definitions for modal sentential calculus and all sentence letters have the same superscript. A wff containing a sentence letter with superscript i (and so, only sentence letters with superscript i) is said to be of *degree i*.

3 *Description of the method* We describe the method as applied to a set of wffs (the initial sentences) to test for consistency. The initial sentences are taken to be all of degree 0. The rules of inference for the non-modal logical constants are as in [1] with, of course, the addition of the superscripts. For example, if we use ϕ^i and Ψ^i to range over wffs of degree i, then the rule for $(\supset)$ is:

$$(\supset): \quad \phi^i \supset \Psi^i$$
$$\diagup \quad \diagdown$$
$$\sim\phi^i \qquad \Psi^i$$

and for $(\sim v)$ is:

$$(\sim v): \quad \sim(\phi^i \vee \Psi^i)$$
$$\mid$$
$$\sim\phi^i$$
$$\sim\Psi^i$$