504
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SEMANTICS FOR S4.03

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Modal system S4.03, as shown in [2], is axiomatized by appending

I1 *ALCLpqCLMLqp*

to some base for S4 containing a primitive rule of necessitation. The purpose of this paper is to provide semantics for this system and its corresponding non-Lewis counterpart, K1.1.5, which is also introduced in [2]. The methods, lemmata, and terminology which we shall employ are taken from Hughes and Cresswell in [3], pp. 150-159.

In [3], p. 74, Hughes and Cresswell define an S4-model as an ordered triple $\langle W, R, V\rangle$, where W is a set of possible worlds, R is a reflexive and transitive accessibility relation holding among the members of W, and V is a value assignment satisfying the conditions stated in [3], p. 73. Now in order to construct a model for S4.03, we need only impose the additional stipulation that the accessibility relation in an S4-model be what we shall call "disjunctively symmetrical." We say that R is disjunctively symmetrical iff for every $w_i \in W$ there exists a w_j such that $w_i R w_j$ and for any $w_k, w_l \in W$ if $w_i R w_k$ and $w_j R w_l$, then either $w_k R w_i$ or $w_l R w_k$. Since modal system S4.03 is a proper extension of S4, we can demonstrate the soundness of our interpretation by simply showing that **I1** is S4.03-logically true. This is accomplished in the following way.

Assume for the sake of reductio that $V(ALCpqCLMLqp, w_i) = 0$. Clearly it follows that

(1) $$V(LCLpq, w_i) = 0$$

and

(2) $$V(CLMLqp, w_i) = 0.$$

From (2) we obtain

(3) $$V(LMLq, w_i) = 1$$
(4) $$V(p, w_i) = 0.$$

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