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THE LOGIC OF CLOSED CATEGORIES

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0 *Introduction* In this paper,* we continue our study, initiated in [4, 5, 10] and reformulated in [11], of the connection between syntactic and semantic criteria for the "equivalence" and "normality" of formal proofs in intuitionist Gentzen systems. As in [11], we interpret proofs as morphisms in free closed categories. But by no longer requiring that these categories are "cartesian", we obtain a coarser equivalence relation than that in [11], still admitting a "reducibility relation" with the Church-Rosser property. As a by-product of this analysis, we are able to obtain necessary and sufficient conditions for the commutativity of diagrams in free closed categories.

1 *Closed categories* The theory of "closed categories" serves as a generalization for categories such as sets, R-modules over a commutative ring R, compactly generated Hausdorff spaces, small categories, etc., in which any two objects have a "tensor product" and in which the "hom-sets" themselves are again sets, R-modules, compactly generated Hausdorff spaces, small categories, etc. Formally, a *closed category* is a list $\langle \mathfrak{K}, \wedge, \supset, \mathrm{I}, \alpha, \lambda, \sigma, \Omega \rangle$ consisting of the following data:

(i) a category $\mathfrak{K}$;
(ii) a bifunctor $\wedge$: $\mathfrak{K} \times \mathfrak{K} \to \mathfrak{K}$ (called "tensor product");
(iii) a bifunctor $\supset$: $\mathfrak{K}^{\mathrm{op}} \times \mathfrak{K} \to \mathfrak{K}$ (called "internal hom");
(iv) a distinguished object I (called the "unit" of the tensor product);
(v) coherent natural isomorphisms α, λ, and σ with components

$$\alpha(A, B, C): A \wedge (B \wedge C) \to (A \wedge B) \wedge C,$$
$$\lambda(A): \mathrm{I} \wedge A \to A,$$

and

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