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INFINITE SERIES OF REGRESSIVE ISOLS UNDER ADDITION

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1 *Introduction* Let E denote the collection of all non-negative integers (numbers), Λ the collection of all isols, Λ_{R} the collection of all regressive isols, and Λ_{TR} the collection of T-regressive isols. (T-regressive isols were introduced in [4].) We recall the definition of an infinite series of isols, $\sum_{\mathsf{T}} a_n$, where $\mathsf{T} \in \Lambda_{\mathsf{R}} - E$ and a_n: $E \to E$:

$$\sum_{\mathsf{T}} a_n = \mathrm{Req} \sum_{0}^{\infty} j(t_n, \nu(a_n))$$

where $j(x, y)$: $E^2 \to E$ is a one-to-one recursive function, t_n is any regressive function ranging over a set in T, and for any number n, $\nu(n) = \{x \mid x < n\}$. Infinite series of isols were introduced by J. C. E. Dekker in [2], where it was shown that $\sum_{\mathsf{T}} a_n \in \Lambda$. In [1] J. Barback studied infinite series of the form $\sum_{\mathsf{T}} a_n$ where $\mathsf{T} \leqslant^* a_{n-1}$. The relation $\mathsf{T} \leqslant^* a_{n-1}$ means that for any regressive function t_n ranging over a set in T, the mapping $t_n \to a_{n-1}$ has a partial recursive extension. Professor Barback proved that for $\mathsf{T} \leqslant^* a_{n-1}$, $\sum_{\mathsf{T}} a_n \in \Lambda_{\mathsf{R}}$. Because

$$a_n \text{ recursive} \Rightarrow \mathsf{T} \leqslant^* a_n \Rightarrow \mathsf{T} \leqslant^* a_{n-1}$$

but not conversely, there are several conditions of varying strength on the function a_n such that $\sum_{\mathsf{T}} a_n \in \Lambda_{\mathsf{R}}$. It is also known [5] that $\mathsf{T} \leqslant^* a_{n-1}$ is not a necessary condition for $\sum_{\mathsf{T}} a_n$ to be a regressive isol.

The following questions were posed by Professor Barback. Let $\mathsf{T} \in \Lambda_{\mathsf{R}} - E$ and let a_n, b_n: $E \to E$ be functions such that $\sum_{\mathsf{T}} a_n, \sum_{\mathsf{T}} b_n \in \Lambda_{\mathsf{R}}$:

(1) Does $\sum_{\mathsf{T}} a_n + \sum_{\mathsf{T}} b_n \in \Lambda_{\mathsf{R}}$?
(2) Does $\sum_{\mathsf{T}} a_n + \sum_{\mathsf{T}} b_n = \sum_{\mathsf{T}} (a_n + b_n)$?

The present paper provides some partial answers to these questions.

2 *Some results* We will assume throughout that $\mathsf{T} \in \Lambda_{\mathsf{R}} - E$ and that a_n, b_n: $E \to E$ with $\sum_{\mathsf{T}} a_n, \sum_{\mathsf{T}} b_n \in \Lambda_{\mathsf{R}}$.

Theorem 1 *Let α and β be disjoint recursive sets with $\alpha \cup \beta = E$ such that*

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