Notre Dame Journal of Formal Logic
Volume XVIII, Number 2, April 1977
NDJFAM

TWO NOTES ON RECURSIVELY ENUMERABLE VECTOR SPACES

RICHARD GUHL

1 *A Characterization of Recursive Spaces* We retain the terminology of [4] of which this note is a continuation.

Definition Let $\sigma_0 = \sigma - (0)$. Let $n \subset \overline{U}_0$. Define $\mathrm{m}(n)$ to be the (largest prime divisor of $n + 1$).

Obviously:

(i) $\mathrm{m}(n)$ *is a partial recursive function,*
(ii) $\mathrm{m}(V_0)$ *infinite iff* $\dim V$ *infinite,*
(iii) m *respects inclusion on sets,*
(iv) m *maps (proper) subspaces to (proper) subsets,*
(v) *for* $\beta \subset \overline{U}_0$, $\mathrm{m}(x)$ $1 - 1$ *on* β *implies* β *is a repère.*

Definition Let V be a subspace of $\overline{U}$. γ is a cobasis for V if γ is a basis for a complementary space for V. η_V is the canonical cobasis for V if η_V is a cobasis for V and $\eta_V \subset \eta$.

Remark The canonical cobasis for a space V is defined to be the set γ such that $\gamma = (e_i \text{ in } \eta | e_i \text{ is not in } (\mathrm{L}(e_j)\ j < i) + V)$.

Proposition F *The canonical cobasis for a recursive space is recursive.*

Proof: Let $f(i)$ list the canonical cobasis in increasing order. If f is a finite function, then its range is recursive. Otherwise f is a recursive function by the Corollary to Proposition C.

Proposition G *For any space* V, $e_n \in \mathrm{m}(V_0)$ *iff* e_n *is not in the canonical cobasis for* V.

Proof: It suffices to show that

(19) $e_n \in \mathrm{m}(V_0)$ iff $e_n \in (\mathrm{L}(e_i)\ i < n) + V)$.

Assume the left hand side. Then there is an element e in V such that:

$$e = r_0e_0 + \ldots + r_ne_n, \text{ where } r_n \neq 0.$$

Received February 28, 1975