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A TABLEAU SYSTEM FOR PROPOSITIONAL S5

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We present an amusing semantic tableau system for propositional S5 which is actually quite efficient in practice. We assume the reader is familiar with the propositional tableau system using unsigned formulas as presented in [4], Ch. II. We continue the α, β classification of formulas, and add two new categories, necessaries (ν) and possibles (π). These, together with their respective components ν_0 and π_0 are defined by the following tables:

ν	ν_0
$\Box X$	X
$\sim\Diamond X$	$\sim X$

π	π_0
$\Diamond X$	X
$\sim\Box X$	$\sim X$

We begin with a tableau system for propositional S4. To the α and β rules of [4] we add the following two rules:

Rule ν: $\dfrac{\nu}{\nu_0}$

(i.e., if a ν formula occurs on a branch, ν_0 may be added to the end of the branch).

Rule π: $\dfrac{\pi}{\pi_0}$ proviso

(interpreted similarly) where the proviso reads: before adding π_0 to the end of a branch on which π occurs, *cross out all formulas on that branch which are not ν formulas*. (Note: a given occurrence of a non-ν formula X may be common to several branches, and it may be desired to cross it out on only one branch. If this happens, simply add fresh occurrences of X to the ends of all branches on which it should remain undeleted.) Now a branch is called closed if it contains X and $\sim X$, both un-crossed out. The above system is propositional S4. There is a completeness proof for the corresponding first order system in [2]. To modify the above into a propositional S5 system we add one more rule:

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