Notre Dame Journal of Formal Logic
Volume XVIII, Number 2, April 1977
NDJFAM

RECURSIVE AND RECURSIVELY ENUMERABLE MANIFOLDS. I

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Foreword In [1] I have presented a sketch of the *Local Recursive Theory*—a generalization of the Recursive Theory, which is quite different from other generalizations: instead of being a study in definability (as, for example, [6] of Platek), or a concrete interpretation (as the Metarecursive Theory of Kreisel-Sacks in [7]), or an abstract axiomatization (as the Theory of Uniformly Reflexive Structures of Wagner in [8]), Local Recursive Theory is the study of sets which admit a *local recursive structure*; this structure is induced via appropriate enumerations of *local neighborhoods* and an effective patching of such neighborhoods.

Local Recursive Theory, or the *Theory of Recursive and Recursively Enumerable Manifolds*, is a further development of the *Theory of Enumerations*, of an integral part of the Recursive Theory, which was systematically studied by Malcev and his students, especially by Yu. Ershov; in [1] I presented a first draft for such a development, considering only a very special case (of injective local enumerations). Here, I develop the Local Recursive Theory in its full generality and in many directions which were not even mentioned in [1].

With the exception of a few pages, the material of this monograph has not been published previously. The monograph was drafted for a course in Generalized Recursive Theory, at the Graduate School of Mathematics at the University of Notre Dame in the first semester of 1974/1975 year.

CHAPTER I—BASIC NOTIONS

Every map $u: N \to U$ of the set N of non-negative integers onto an at most denumerable, non-empty set U, is called an *enumeration* of U; if it is bijective it will be called an *indexing* of U. Using enumerations we can extend recursive notions to any enumerated set U. For example, a map $f: U \to U$ of U into U will be called *u-recursive* iff (if and only if) there is an r. (recursive) function $f^*: N \to N$, such that, for all $n \in N$,

$$f(u(n)) = u(f^*(n)), \tag{1.1}$$

Received July 19, 1975