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# SIMPLIFIED FORMALIZATIONS OF FRAGMENTS OF THE PROPOSITIONAL CALCULUS

ALAN ROSE

Henkin has given [1] a general method of formalizing 2-valued propositional calculi whose primitive functors are such that material implication is definable in terms of them. Let the primitive functors, other than implication if implication is a primitive functor, be the functors $F_i$ of $n_i$ arguments $(i = 1, \ldots, b)$ and let the formulae $P_1, \ldots, P_{n_i}, F_iP_1 \ldots P_{n_i}$ take the truth-values $x_1, \ldots, x_{n_i}, f_i(x_1, \ldots, x_{n_i})$ respectively $(i = 1, \ldots, b)$. The axiom schemes are

*A1* $CPCQP$,
*A2* $CCPQCCPCQRCPR$,
*A3* $CCPRCCCPQRR$,
*A4* $CV_{x_1}P_1Q \ldots CV_{x_{n_i}}P_{n_i}QV_yF_iP_1 \ldots P_{n_i}Q(y = f_i\,(x_1, \ldots, x_{n_i});$
$x_1 = \mathbf{T}, \mathbf{F}; \ldots; x_{n_i} = \mathbf{T}, \mathbf{F}; i = 1, \ldots, b)$,

*A4* denoting $\sum_{i=1}^{b} 2^{n_i}$ axiom schemes and the functors $V_{\mathbf{T}}$, $V_{\mathbf{F}}$ being defined by the equations

$$V_{\mathbf{T}}PQ =_{df} CCPQQ,$$
$$V_{\mathbf{F}}PQ =_{df} CPQ.$$

The only primitive rule of procedure is

*R1* *If* $P$ *and* $CPQ$ *then* $Q$.

We shall show how to reduce[1] the number and lengths of the axiom schemes.

It follows at once from a result of Łukasiewicz [3] that *A1-3* may be replaced by the axiom scheme

*B1* $CCCPQRCCRPCSP$.

1. The axiom schemes *C* are similar to those obtained by using a general method of Shoesmith [5], but his completeness proof is non-constructive.

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