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THE Q-CONSISTENCY OF $\mathcal{F}_{22}$

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In his [CSC],[1] Curry proved the consistency of a system, which he there defines and calls $\mathcal{F}_{22}$, and which is closely related to the system $\mathcal{F}_{21}^{*}$ of [CLg.II] §15C.[2] This is essentially a type-free intuitionistic predicate calculus without conjunction, alternation, or negation but with quantification over propositions and propositional functions. However, Curry's consistency proof is rather weak, since it only proves that every theorem of the system belongs to a class of obs (terms) which are defined to be canonical (called *canobs*) and since the canobs are those obs which are to be interpreted as propositions this proof leaves open the possibility that every ob which is to be interpreted as a proposition is a theorem of the system.

In this paper,[3] a stronger consistency result is proved. This is done by proving the elimination theorem (Gentzen's *Hauptsatz*). From this it follows that if an atom (atomic ob, or atomic term) **Q** is introduced in a natural way to represent equality, then $\vdash \mathbf{Q}XY$ holds if and only if $X = Y$ in the underlying C-system (i.e., if and only if X is convertible to Y using the conversion rules of the underlying system of combinatory logic or λ-conversion). Since it can be shown that $\mathbf{S} \neq \mathbf{K}$, it will follow that $\vdash \mathbf{QSK}$ does not hold, and hence there is an ob, **QSK**, which is interpreted as a proposition ($\mathbf{S} = \mathbf{K}$) but which is not a theorem.

In addition, an error in the theory of canobs in [CLg.II] §15B3 will be corrected here.

1 *The theory of canobs* Since the theory of $\mathcal{F}_{22}$ depends heavily on the theory of canobs, I will begin with the latter.

The error in the theory of canobs of [CLg.II] §15B3 occurs in Lemma 3.3. In the proof of that lemma, it is established that

(1) $$Yx_1 \ldots x_n \succ Z,$$

and it is claimed that from this, by property (ξ), it follows that

(2) $$Y \succ \lambda x_1 \ldots x_n . Z.$$

But this holds only if the underlying system is synthetic.[4] If the underlying

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