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A THEORY OF RESTRICTED VARIABLES WITHOUT EXISTENCE ASSUMPTIONS

MARGARET MURPHY PRULLAGE

1 *Introduction* The individual variables of ordinary first-order logic are generally thought of as ranging over all the objects in a certain set, the universe or domain of discourse, with no discrimination made among the variables. In everyday mathematics, however, this is often not the case, and some variables, usually distinguished by the use of different letters, are restricted in their signification to some proper subset of the domain of discourse. For example, the letters x, y, and z may refer to real numbers in a formula such as $x^2 - y^2 = (x + y)(x - y)$, but there may also be formulas of the sort "For all integers m, ..." or "There is a positive prime p, such that . . .." Thus it is useful to formulate a logic which allows for the restriction of variables to certain ranges as well as for the general interpretation of variables.*

Bourbaki, in his treatment of logic in [1], allows for restricted quantification by defining quantifiers $\exists_A x$ and $\forall_A x$ in terms of the existential quantifier $\exists$, $(\exists_A x)R$ being defined as $(\exists x)(A \mathbin{\&} R)$. Intuitively, if A and R are formulas expressing properties of x, then $(\exists x)(A \mathbin{\&} R)$, meaning "There is an x, such that A and R hold," is equivalent to $(\exists_A x)R$, interpreted as "There is an x of kind A, such that R holds." In $(\exists_A x)R$, x is restricted to objects satisfying A by the symbol $\exists_A$. $(\forall_A x)R$ is defined to be $\neg(\exists_A x)\neg R$. The symbols $\exists_A$ and $\forall_A$ might be used in a demonstration if one is interested only in objects satisfying A, where A might express the property of being an integer, or a positive prime. In [13], Rosser discusses restricted variables in some detail; his approach appears to differ from Bourbaki's since he considers restricted variables rather than restricted quantifiers. He uses Greek letters to refer to restricted variables; α might

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