Notre Dame Journal of Formal Logic
Volume XVII, Number 4, October 1976
NDJFAM

METACOMPLETENESS

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In [1], a logic was called *coherent* provided that it could plausibly be interpreted in its own metalogic. By deepening and making more intuitive the logical analysis implicit in [1], we develop here a kindred notion of *metacompleteness*; a logic is metacomplete provided that *exactly* the sentences true on a certain preferred interpretation of that logic in its metalogic are theorems. Acquaintance with [1] is not presupposed. We shall show in particular that a number of familiar logics, e.g., of the intuitionist, modal, and relevant families, are metacomplete, and that accordingly these logics share with intuitionist calculi two interesting properties:

(A) If $A \vee B$ is a theorem, so is at least one of A or B.
(B) If $\exists x A(x)$ is a theorem, so is some substitution instance $A(t)$, for some term t.

Harrop, Rasiowa, and Kleene have found simple truth-functional-style arguments for (A) and (B) in the case of intuitionist theories in particular, as Kripke has called to my attention. Here, by building on the techniques of [1], we present such results in a wider context, applicable in particular to the relevant logics whose theory is set out in [5].[1]

Negation was treated classically in [1], but our main interest here will be in logics that are either negation-free or which formalize a negation acceptable from a generally constructive point of view. This is less restrictive than it sounds, in particular for the relevant logics, for which some of our results are new, since the methods of [2] may be applied to show, at least in their sentential parts, their negation-free theorems captured by their negation-free axioms and rules; the result is that the relevant logics satisfy the motivating conditions (A) and (B) in their positive parts.[2] As we shall note for the system **R** of relevant implication in particular, if the classical negation axioms for **R** of Anderson and Belnap are replaced with intuitionistically acceptable ones, getting a system **RJ**, (A) and (B) hold throughout.

Received November 26, 1971