446
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TWO NOTES ON ACKERMANN'S SET THEORY

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We give solutions to two problems which concern Ackermann's set theory, **A**.* This theory was introduced in [1] and it is now formulated in the first-order predicate calculus with identity, using ϵ for membership and an individual constant, V, for the class of all sets. We use the letters ϕ, Ψ, θ, and χ to stand for formulae which do not contain V and capital Greek letters to stand for any formulae. Then the axioms of **A** are the universal closures of

A1 $\forall t\,(t \epsilon x \leftrightarrow t \epsilon y) \rightarrow x = y$,
A2 $\exists z \forall t\,(t \epsilon z \leftrightarrow t \epsilon V \wedge \Theta)$,
A3 $x \epsilon V \wedge (t \epsilon x \vee \forall u\,(u \epsilon t \rightarrow u \epsilon x)) \rightarrow t \epsilon V$,
A4 $x, y \epsilon V \wedge \forall t(\Psi(x, y, t) \rightarrow t \epsilon V) \rightarrow \exists z \epsilon V \forall t\,(t \epsilon z \leftrightarrow \Psi(x, y, t))$,

where all free variables are shown in *A4* and z does not occur in the Θ of *A2*. **A*** is **A** augmented by the axiom

A5 $x \epsilon V \wedge \exists u\, u \epsilon x \rightarrow \exists u \epsilon x\, \forall t \epsilon u\; t \notin x$.

Firstly, we shall solve a problem from [3], by extending some of the work on permutation models (see [2], for instance) to models of **A**.

Definition 1 A functional formula $y = F(x)$ is said to be a permutation if it represents a bijection of the universe onto itself. If $y = F(x)$ is a permutation then we write $x \epsilon_F y$ for $F(x) \epsilon y$ and Ψ_F for the formula Ψ with all instances of ϵ replaced by ϵ_F.

Theorem 2 *If $y = F(x)$ is a functional ϵ-formula such that*

(i) *F is a permutation,*
(ii) *$x \epsilon V$ iff $F(x) \epsilon V$,*

then we can interpret **A** *in* **A** *using ϵ_F for the membership relation and V for V.*

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