

NOTE ON AN INDEPENDENCE PROOF OF JOHANSSON

E. A. NEMESSZEGHY

In [1], p. 124, I. Johansson proves that the propositional formula $\neg\neg(\neg\neg a \supset a)$ is underivable in his minimal logic. He establishes this result by the well-known matrix-method: he gives certain matrices in which all the axioms of the minimal logic are valid, the rules of the system preserve validity, but $\neg\neg(\neg\neg a \supset a)$ is invalid. The matrices he uses are 5×5 matrices, i.e., of 5 rows and 5 columns for the binary connectives. The purpose of this short note is to point out that there are simpler 3×3 matrices which do the same job. The matrices for the connectives are given below. The only designated value is 1.

\supset	1	2	3	\wedge	1	2	3	\vee	1	2	3	x	$\neg x$
*1	1	2	3	*1	1	2	3	*1	1	1	1	*1	2
2	1	1	3	2	2	2	3	2	1	2	2	2	1
3	1	1	1	3	3	3	3	3	1	2	3	3	1

It is easy to check that all the axioms of the minimal logic are valid in these matrices, and the rules of the system preserve validity; yet $\neg\neg(\neg\neg a \supset a)$ is invalid, for if the value of ' a ' is 3 then $\neg\neg(\neg\neg 3 \supset 3) = \neg\neg(2 \supset 3) = \neg\neg 3 = 2 \neq 1$.

REFERENCE

- [1] Johansson, I., "Der Minimalkalkül, ein reduzierter intuitionistischer Formalismus," *Compositio mathematica*, vol. 4 (1936), pp. 119-136.

Heythrop College, University of London
London, England

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