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A SHORT EQUATIONAL AXIOMATIZATION OF ORTHOMODULAR LATTICES

BOLESŁAW SOBOCIŃSKI

By definition, *cf.* e.g., [2], p. 53, an orthomodular lattice is an ortholattice satisfying the following formula:[1]

K1 $[ab]: a, b \in A . a \leq b .\supset. a \cup (a^{\perp} \cap b) = b$

In this note it will be proved that:

(**A**) *Any algebraic system*

$$\mathfrak{A} = \langle A, \cup, \cap, {}^{\perp}\rangle$$

where $\cup$ *and* $\cap$ *are two binary operations and* $^{\perp}$ *is a unary operation defined on the carrier set* A, *is an orthomodular lattice, if it satisfies the following three mutually independent postulates:*

C1 $[abcd]: a, b, c, d \in A .\supset. a \cup ((a \cup ((b \cup c) \cup d)) \cap a^{\perp}) = ((d^{\perp} \cap c^{\perp})^{\perp} \cup b) \cup a$
C2 $[ab]: a, b \in A .\supset. a = a \cup (b \cap b^{\perp})$
C3 $[ab]: a, b \in A .\supset. a = a \cap (a \cup b)$[2]

Proof of (**A**):

1 Clearly, postulates *C2* and *C3* are the theses of any ortholattice. It remains to prove that, in the field of an arbitrary ortholattice, *C1* is inferentially equivalent to formula *K1*.

1.1 First, we shall prove that in the field of any lattice *K1* is inferentially equivalent to formula *R1* given below.

1.1.1 Assume **L**. Then we have at our disposal:

1. Throughout this paper A indicates an arbitrary but fixed carrier set, **L** a lattice, and **OL** an ortholattice. The so-called closure axioms are assumed tacitly.

2. Of course, in this postulate-system the operations $\cup$, $\cap$ and $^{\perp}$ are not mutually independent.

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