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PLEDGER LEMMA AND THE MODAL SYSTEM S3°

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1 In [8] I defined modal systems S3.02, S3.03, and S3.04 as the systems which are obtained by adding to S3 the respective axioms

Ł1 $\mathfrak{C}\mathfrak{C}\mathfrak{C}pLppCLMLpp$
Ł2 $\mathfrak{C}\mathfrak{C}\mathfrak{C}pLpp\mathfrak{C}LMLpp$
L1 $\mathfrak{C}LMLpCpLp$

Remark: It should be noted that either **Ł1** or **Ł2** can be accepted as a proper axiom of S4.02, *cf.* [6], and that **L1** is a proper axiom of S4.04, *cf.*, e.g., [9]. Obviously, these axioms are not consequences of S4.

1.1 In [8] it has been established:

(a) that each of the systems S3.02, S3.03, and S3.04 is a proper extension of S3 and that they do not contain S4.
(b) that system S3.04 is a subsystem neither of S3.02 nor of S3.03.

and

(c) that S3.02 is a subsystem of S3.03.

On the other hand, in [8] the following problems were left open:

(d) is S3.02 a proper subsystem of S3.03?

and

(e) does S.04 contain S3.02 or S3.03?

1.2 In [4] G. F. Schumm solved problem (d), proving metalogically that in the field of S3 axiom **Ł1** implies **Ł2**, and, therefore, S3.02 = S3.03. Independently, in [3], K. E. Pledger obtained the same result, but used, in some respects, a different method. Namely, he remarked that it is easy to prove metalogically that the following formula (called here the Pledger lemma):

PL $\mathfrak{C}\mathfrak{C}LpCLqr\mathfrak{C}Lp\mathfrak{C}Lqr$

is a thesis of system S3. Hence, it follows immediately from this fact that

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