

AXIOM SETS EQUIVALENT TO SYLLOGISM AND PEIRCE

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We consider the implicational propositions 1. $CCRCpqp$, 2. $CCqrCCpqCpr$, 3. $CCpqCCqrCpr$, 4. $CCCrCpqp$, 5. $CCCpqp$, 6. $CpCqp$, 7. $CPCQP$, where capitalized variables stand for arbitrary implications. In [1], pp. 173-174, C. A. Meredith showed the inferential equivalence of the sets (2, 4), (3, 4), (3, 5, 6) and that (3, 5) yields 2 and (Thomas) 7. We show that inferentially equivalent are (1, 2), (1, 3), (3, 5).

A. (1, 2) yields 3 and 5. *Proof:* If in Meredith's deduction of (3, 4) from (2, 4) we change the last proof line from DD2.18.15 to DD.2.18.17 the resulting seventeen proof lines produce 3 from our (1, 2), with 5 turning up after the first five detachments.

B. (1, 3) yields 2. *Proof:* (1, 3) yields 5, since DDDD33311 = 5. (3, 5) yields 2 (Meredith).

C. (3, 5) yields 1. *Proof:* (3, 5) yields 7 (Thomas) so by 3 we have CCCQPrCPr, whence by substitution $CCRCpqpCCpqp$, from which 3 and 5 yield 1.

A, B, C prove the theorem. As a basis for this system (1, 2) appears to develop by far the most quickly and simply.

REFERENCE

- [1] Meredith, C. A., and A. N. Prior, "Axiomatics of the propositional calculus," *Notre Dame Journal of Formal Logic*, vol. IV (1963), pp. 171-187.

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