ON SOME MODAL LOGICS RELATED
TO THE Ł-MODAL SYSTEM

ROBERT L. WILSON

1 *Introduction* Five modal logics are introduced in this paper. They are denoted by **F*F** where **F** = **Ł, W, S, D** and **E**. **F*** denotes the semantics (see section **3**) and **F** denotes the formal system (see section **4**). Each modal logic **F*F** is composed of four sub-logics $\mathbf{F_i^*F_i}$ (i = **1, 2, 3, 4**) corresponding to four different kinds of provability and rejection, namely $\mathbf{F_i}$-provability and $\mathbf{F_i}$-rejection.

Since the idea of these modal logics arose from certain semantical considerations rather than from formal ones, some questions on the semantics of the Ł-modal system and 3-valued logic are mentioned in section **2**. These questions help to provide the motivation for the semantics **F*** and a semantics $\mathbf{Ł_3^*}$ for the Ł-modal system in particular.

The formal treatment uses an adaption of Smullyan's method of the analytic tableaux [6] and is illustrated for **Ł** in section **4**. In section **5**, the semantical consistency and completeness proofs for **Ł*Ł** are given. The sub-systems $\mathbf{F_1}$, $\mathbf{Ł_2}$, $\mathbf{W_2}$, $\mathbf{S_2}$ violate some of the laws of Łukasiewicz's basic modal logic [1]. Halldén's incompleteness property [5] holds in the sub-systems $\mathbf{F_3}$. Also, the sub-systems $\mathbf{F_4}$ are formally inconsistent (see section **7**). The connection between all these formal properties and the underlying semantics is discussed in section **7**.

2 *Some questions and comments on the Ł-modal system and 3-valued logic*

Question 1 Considering Łukasiewicz's four truth-values underlying his semantics for the Ł-modal system, what do the four truth-values mean?

Comment It is interesting to note that when Łukasiewicz is referring to the semantics in [1], [2], he is basically talking in a 2-valued idiom, i.e., he simply uses the words 'true' and 'false' (*cf*. Łukasiewicz's truth-values '1' and '4'). Concerning the values '2' and '3', Łukasiewicz in his paper [2] refers to them as "denoting possibility, but nevertheless both values represent one and the same possibility in two different shapes."