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NEGATION DISARMED

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The purpose of the present note is to extend the methods of [1] to show, for several interesting systems of quantificational logic, that their negation-free fragments are determined by their negation-free axioms, even in the presence of strong classical negation axioms. Among these systems, as in [1], are the relevant logics of Anderson and Belnap, presented here in their first-order versions **RQ**, **EQ**, etc. We generalize the results of [1] to the extent that they apply here not merely to positive logics $\mathbf{L}^+$ but to positive $\mathbf{L}^+$-theories; i.e., it turns out for the relevant logics (and some others) that the set of negation-free theorems of a first-order theory all of whose non-logical axioms are negation-free is completely determined on applying negation-free logical axioms and rules to these non-logical axioms.

Aside from its intrinsic interest, the point of this result lies in the fact that the negation-free part of the relevant logics is intuitionistically acceptable, though its negation axioms are not. This acceptability extends to possession of certain interesting structural properties, e.g., if $A \vee B$ is a negation-free theorem of one of the relevant logics, so is at least one of A and B, as was noted at the sentential level in [2]; similarly, as is to be shown in a paper in preparation, if $\exists x A(x)$ is a negation-free theorem, so is an instance $A(t)$ for some term t; both properties, of course, are intuitionistic. What we want to show, accordingly, is that there are no theorems in the constructively acceptable negation-free parts of the relevant logics that are only provable by constructively unacceptable methods, i.e., by detours through the properties of classical negation. (The point is unlikely to be missed, but the claim is that relevant logics have certain *formal* properties that are intuitionistically acceptable; as is usual in these matters, no such claim is entered for the *informal* arguments employed to establish this result.)

The reader is presumed to have access to [1], and so its methods and terminology are used freely. Arguments, by and large, are old, being adapted here only as is necessary for the richer context. References to axioms (e.g., A1), etc., are to [1].

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