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# SOME NOTES ON "A DEDUCTION THEOREM FOR RESTRICTED GENERALITY"

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In [2] the deduction theorem for $\Xi$:

*If* $X_0$, $X \vdash Y$, *and* $X_0 \vdash \mathbf{L}([\mathbf{u}]\mathbf{X})$ *where* $u$ *is not involved in* $X_0$, *then* $X_0 \vdash X \supset_u Y$,[1]

was proved using the following axioms.

Axiom 2. $\vdash \mathbf{L}x \supset_x \Xi xx$.
Axiom 3. $\vdash \mathbf{L}x \supset_{x,y} : xu \supset_u . \ yuv \supset_v xu$.
Axiom 4. $\vdash \mathbf{L}x \supset_{x,t} : xu \supset_u yu(tu) \supset_y . \ (xu \supset_u (yuv \supset_v zuv)) \supset_x (xu \supset_u zu(tu))$.
Axiom 5. $\vdash \mathbf{L}x \supset_x \Xi x \ (\mathbf{WQ})$.
Axiom 6. $\vdash \Xi \mathbf{IH}$.
Axiom 7. $\vdash \mathbf{LH}$.

Of these, $\vdash \mathbf{LH}$ as it restricts the system to obs which satisfy

$$\mathbf{A}u \vdash \mathbf{H}(\mathbf{H}u),$$

is a somewhat unsatisfying axiom. In particular with $\mathbf{E} = \mathbf{A}$ it is inconsistent with the others (see [1]).

Also the rules obtained by applying Rule $\Xi$ once to each of the remaining axioms are consistent. This was shown in an unpublished paper by H. B. Curry and the author. Curry in [3] proved that for an equivalent system no nonpropositions are provable and Seldin in [4] has shown consistency in a stronger sense.

We show here that the deduction theorem for $\Xi$ can be proved without $\vdash \mathbf{LH}$. We achieve this by taking $\mathbf{L}$ as primitive (rather than as defined by $\mathbf{L} \equiv \mathbf{FAH}$) and we define $\mathbf{H}$ as $\mathbf{BLK}$. Axiom 3 leads to the rule:

$$\mathbf{L}x, \ xu \vdash yuv \supset_v xu$$

1. In [2] $\mathbf{L} \equiv \mathbf{FAH}$. $X \supset_u Y$ is an alternative notation for $\Xi([u]X)([u]Y)$.

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