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ON THE RELATION BETWEEN FREE DESCRIPTION THEORIES AND STANDARD QUANTIFICATION THEORY

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Meyer and Lambert [2] constructed a mapping which takes formulas of free quantification theory into formulas of standard quantification theory and preserves validity. One adds a one-place predicate D to the vocabulary and translates thus:

For atomic P, $\sigma(P) = P$

$$\sigma(A \rightarrow B) = \sigma(A) \rightarrow \sigma(B)$$
$$\sigma(-A) = -\sigma(A)$$
$$\sigma((x)A) = (x)[Dx \rightarrow \sigma(A)].$$

There is also an interesting mapping τ from models of free quantification theory (**FQ**) to models of standard quantification theory (**SQ**). If $\mathfrak{M}$ is a model for **FQ** such that $\mathfrak{M} = \langle D, D^*, R\rangle$, then $\tau(\mathfrak{M}) = \langle D \cup D^*, R, D\rangle$. In other words, the domain of the **SQ** model is the union of the two **FQ** domains, each predicate letter receives the same interpretation as in **FQ** and the predicate letter D is assigned the domain of the **FQ** model. It is easy to show that for any sequence α, α satisfies A in $\mathfrak{M}$ iff α satisfies $\sigma(A)$ in $\tau(\mathfrak{M})$.[1]

One can construct a similar pair of mappings for Scott's free description theory [3], which is obtained by adding to free quantification theory the two schema

I) $(y)[y = \urcorner xA] \leftrightarrow (x)[x = y \leftrightarrow A]$ *where* y *is not free in* A
II) $-(Ey)[y = \urcorner xA] \rightarrow \urcorner xA = \urcorner x(x \neq x)$.

Models of the Scott system are simply models of **FQ** with the further requirement that one specify an element of D^* which is the denotation of all bad descriptions. In order to construct a mapping τ for this system, we

1. Thus the rather lengthy discussion of nominal interpretations in [2] could have been dispensed with since including them gives the same class of valid formulas.

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