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ALGEBRAIC SEMANTICS FOR S2^0 AND NECESSITATED EXTENSIONS

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Algebraic techniques are used to show that Feys' system S2^0 (*cf.* [1]) and certain necessitated extensions of S2^0, such as Lewis' systems S2 and S3, have the finite model property, and accordingly are decidable. Representation theorems are then used to establish set-theoretical semantics for the modal systems studied. Where the results obtained are not new they improve on earlier results (such as those of Lemmon in [3]) in two respects; first they provide direct algebraic treatments of the systems, and second they furnish better semantical results (see the discussion of theorem J for S2). The techniques used however follow those of McKinsey (in [4]) and Lemmon (in [2] and [3]). Since it is now known that these techniques do not work for all necessitated extensions of S2^0, a somewhat piecemeal approach is inevitable. Weak results are also obtained for Feys' system S1^0 and Lewis' system S1 (for details of these systems see [1]).

The sentential systems studied are of interest not so much as systems containing a viable necessity operator '$\Box$', but as intensional logics which axiomatise epistemic or other operators. For instance S2^0 can be interpreted as an epistemic logic such that '$\Box$' reads 'it is believed reasonably that', and S2 as an epistemic logic where '$\Box$' reads 'it is known that'. The set-theoretical semantics established are however independent of these epistemic interpretations.

The basic system examined, Feys' S2^0, has as postulates:

T1. $A \,\&\, B \,-\!\!3\, A$
T2. $A \,\&\, B \,-\!\!3\, B \,\&\, A$
T3. $(A \,\&\, B) \,\&\, C \,-\!\!3\, .A \,\&\, (B \,\&\, C)$
T4. $A \,-\!\!3\, A \,\&\, A$
T5. $A \,-\!\!3\, B \,\&\, B \,-\!\!3\, C \,-\!\!3\, .A \,-\!\!3\, \dot{C}$
T6. $\Diamond(A \,\&\, B) \,-\!\!3\, \Diamond A$

Strict Detachment (**SD**): $\vdash A, \vdash A \,-\!\!3\, B \twoheadrightarrow \vdash B$
Adjunction (**A**): $\vdash A, \vdash B \twoheadrightarrow \vdash A \,\&\, B$
Substitutivity of Strict Equivalents (**SSE**): $\vdash A \,\varepsilon\!\!3\, B, \vdash C(A) \twoheadrightarrow \vdash C(B)$

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